LOGIC IMPLICATION

Recall

A function f is *differentiable* at a if $f'(a)$ exists, i. e. if

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

exists.

If f is differentiable at a then f is continuous at a

f is differentiable at a

$$\Downarrow$$

f is continuous at a

$$P \Rightarrow Q$$

P = « f is differentiable at a »

Q = « f is continuous at a »

$$P \Rightarrow Q$$

P = Student X is in CMC 130 on MW at 11am

Q = Student X is a calculus student

Is this implication true?

YES!

Question:

If P $\Rightarrow$ Q is true, then what can we say about:

not Q $\Rightarrow$ not P

Q $\Rightarrow$ P

P = Student X is in CMC 130 on MW at 11am

Q = Student X is a calculus student

not P = Student X is **not** in CMC 130 on MW at 11am

not Q = Student X is **not** a calculus student

Is it true that: not Q $\Longrightarrow$ not P ?

Yes!

P = Student X is in CMC 130 on MW at 11am

Q = Student X is a calculus student

Is it true that: $Q \Rightarrow P$?

NO!

Counterexample: each student in sections 2,3,4,5,6,7,901 of calculus is a calculus student who is not in CMC 130 on MW at 11am.

More exageration

 $\Longrightarrow$ insect

TRUE

More exageration

not insect $\Rightarrow$ not 

TRUE

More exageration

insect $\Rightarrow$ 

FALSE!

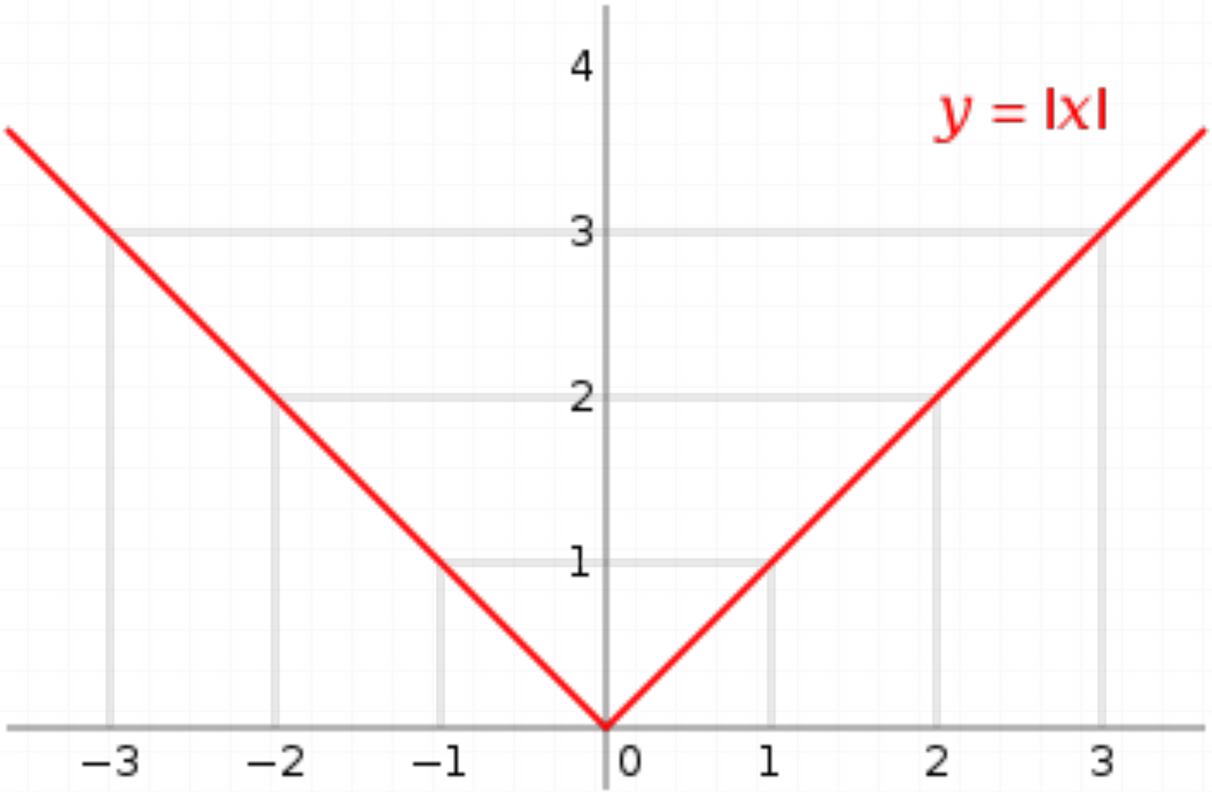
counterexample

f is differentiable at a $\overset{\textbf{T}}{\Longrightarrow}$ f is continuous at a

f is **not** continuous at a $\overset{\textbf{T}}{\Longrightarrow}$ f is **not** differentiable at a

f is continuous at a $\overset{\textbf{F}}{\Longrightarrow}$ f is differentiable at a

Counterexample



$f(x) = |x|$ is continuous at 0, but not differentiable at 0.

Recap!

The implication **$P \Rightarrow Q$** is true when every time the statement P is true, then also the statement Q is true. Hence:

- If you want to show that the implication $P \Rightarrow Q$ is **true**, you need a **proof**;

- If you want to show that the implication $P \Rightarrow Q$ is **false** you need a **counterexample**: this means that you need an example of something that verifies P but does not verify Q (indeed in this case P will be true, while Q will be false).

$$P \Leftrightarrow Q$$

P = The grade of student X is A

Q = The final grade of student X is more than 90%

All the definitions are « if and only if »

Ex: A function f is *continuous* at a if (and only if)

$$\lim_{x \to a} f(x) = f(a)$$