Calculus I - MAC 2311 - Section 001

Review session Final Exam

4/26/2018

Ex 1. Compute the following (definite or indefinite) integrals:

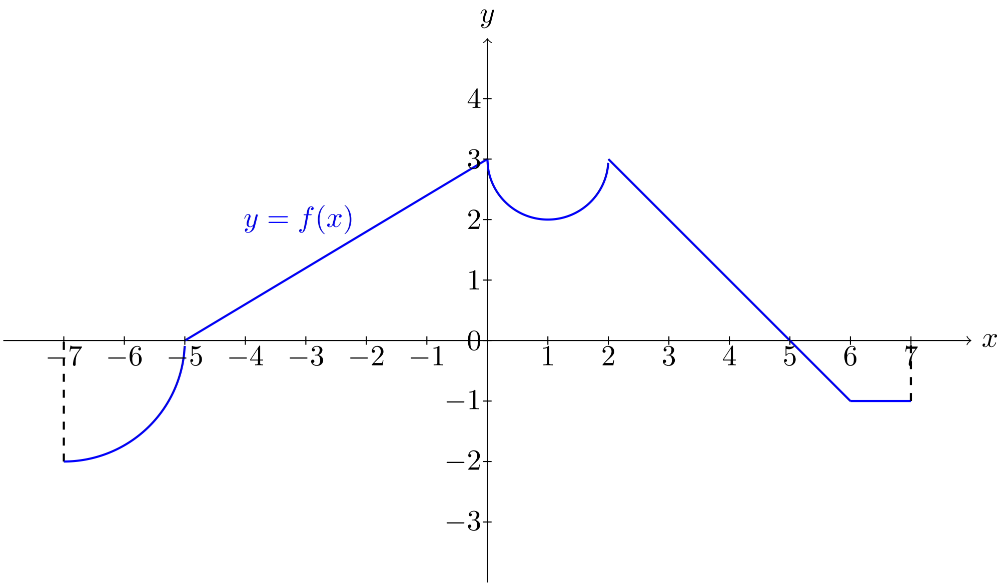
a) $\displaystyle\int 3\sin(x) + \frac{4}{1+x^2} + 2\,dx$

b) $\displaystyle\int (t+3)(2-t^2) + \frac{\sqrt{t}+t}{t^2}\,dt$

c) $\displaystyle\int_{-\pi}^{\frac{\pi}{2}} 3\sin(x) - 8\cos(x)\,dx$

d) $\displaystyle\int_{1}^{0} -2e^u + \frac{1}{1+u^2}\,du$

Ex 2. Let f be the function whose graph is the following:



a) Compute $\displaystyle\int_{-7}^{7} f(x)\,dx$.

b) Compute $\displaystyle\int_{-7}^{0} 3f(x)\,dx + \int_{0}^{5} f(x) + \sqrt{25-x^2}\,dx - \int_{7}^{5} 2f(x) + 2x\,dx$.

Ex 3. A particle is moving with acceleration given by the function $a(t) = 12t^2 + 2\sin(t)$ (measured in meters per second squared).

a) Find the position function of the particle if its initial velocity is 5 meters per second and the position at $t = \pi$ is π^4 meters.

b) Find the position function of the particle if its initial position is 2 meters and its position at $t = \frac{\pi}{2}$ is 0 meters.

Ex 4. Compute the derivative of the following functions:

a) $f(t) = \sqrt{1 + t\arccos(t)}$

b) $f(x) = \dfrac{e^{\tan(x)} + 1}{\cos(x)}$

c) $f(s) = \arctan(\sqrt{s}) \cdot \ln(2s)$

d) $f(t) = (\sin(t))^{t^2}$

e) $g(x) = \displaystyle\int_{-1}^{x} \ln(t^2 + 1)dt$

f) $g(t) = \displaystyle\int_{0}^{e^t} \frac{x^2 - 1}{x^2 + 1}dx$

g) $g(s) = \displaystyle\int_{\cos(s)}^{3s} \sin(t^2 + 1)dt$

Ex 5. A cone shaped paper drinking cup is to be made to hold $27\,\mathrm{cm}^3$ of water. Find the height and radius of the cup that will require the least amount of paper.

Ex 6. Compute the following limits:

a) $\displaystyle\lim_{x\to-\infty} -x^4 + x^2$

Does the function $f(x) = -x^4 + x^2$ have a horizontal asymptote at $-\infty$? If yes, write its equation.

b) $\displaystyle\lim_{t\to1} \frac{\ln(1 + \ln(t))}{t^2 - 1}$

Is $t = 1$ a vertical asymptote for the function $f(t) = \dfrac{\ln(1 + \ln(t))}{t^2 - 1}$?

c) $\displaystyle\lim_{x\to\infty} \frac{-\sqrt{2}x^5 - 8x^4 + 5}{\pi x^5 - e}$

Does the function $f(x) = \dfrac{-\sqrt{2}x^5 - 8x^4 + 5}{\pi x^5 - e}$ have a horizontal asymptote at ∞? If yes, write its equation.

d) $\displaystyle\lim_{x\to-3} \frac{\cos(\pi x)}{(x + 3)^2}$

Is $x = -3$ a vertical asymptote for the function $f(x) = \dfrac{\cos(\pi x)}{(x + 3)^2}$?

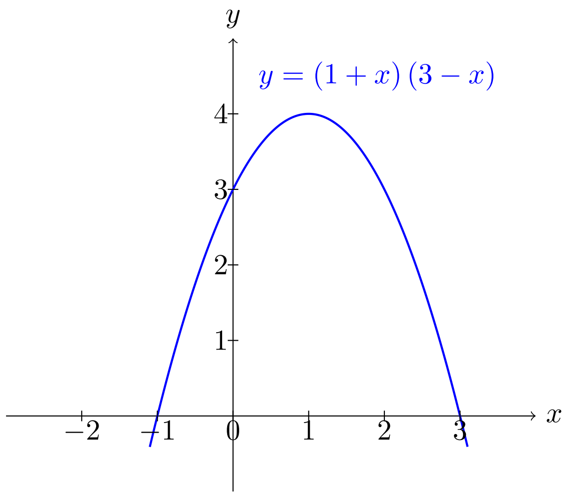
e) $\displaystyle\lim_{x\to0} \frac{e^{x^2} - 1}{3x^2}$

Is $x = 0$ a vertical asymptote for the function $f(x) = \dfrac{e^{x^2} - 1}{3x^2}$?

f) $\lim\limits_{x\to\infty} \displaystyle\int_1^x \frac{1}{1+t^2} + \frac{1}{t^2} dt$

Does the function $g(x) = \displaystyle\int_1^x \frac{1}{1+t^2} + \frac{1}{t^2} dt$ have a horizontal asymptote at ∞? If yes, write its equation.

Ex 7. Consider the function $f(x) = (1 + x)(3 - x)$ whose graph on the interval $[-1, 3]$ is sketched below. Let S be the region between the curve $y = f(x)$, the x-axis and the lines $x = -1$ and $x = 3$.



a) Draw in the picture above the rectangles associate to the right Riemann sum with $n = 4$.

b) Approximate the area of S with the right Riemann sum with $n = 4$.

c) Express the area of S as a definite integral.

d) Compute the exact value of the area of S.

e) Was your approximation an underestimate or an overestimate?

Ex 8 Let $f(x) = x^4 - 4x^2$.

(a) List the following, showing all work:

- the x and y- intercepts, if any
- the horizontal and vertical asymptotes, if any
- the intervals of increase and decrease of f
- all local maximum and local minimum values of f
- the intervals over which f is concave up and the intervals over which f is concave down
- all inflection points

Sketch the graph of f and label all the items that you listed.

(b) Repeat the exercise for the functions $g(t) = \frac{1}{t} + t + 1$ and $h(x) = x^2 e^x$.

Ex 9. Let $f(x) = \cos\left(\tan^{-1}\left(\frac{1}{e^x}\right)\right)$. Simplify the expression of f and compute $f(0)$.

Ex 10. At noon ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 pm.

Ex 11. Using the Mean Value Theorem and the Fundamental Theorem of Calculus, prove that if f is continuous on $[a, b]$ then there exists a number c in (a, b) such that

$$\int_a^b f(x)dx = f(c)(b-a).$$

Give a geometrical interpretation of this result.

Remark: This theorem is called **Mean Value Theorem for Integrals**.

Ex 12. A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is as small as possible?