Calculus I - MAC 2311 - Section 001

Review session Test 2

3/01/2018

Ex 1. Sketch the graph of a function $g(t)$ which satisfies **all** the following conditions:

a) $g'(t) > 0$ for all $t < -2$,

b) $g'(-2)$ does not exist,

c) $g'(t) > 0$ for all t in $(-2, 3)$,

d) $g'(3) = 0$,

e) $g'(t) > 0$ for all $t > 3$.

Ex 2. The **ideal gas law** relates the temperature, pressure, and volume of an ideal gas. Given n moles of gas, the pressure P (in kPa), volume V (in liters), and temperature T (in kelvin) are related by the equation

$$PV = nRT,$$

where R is the molar gas constant ($R \cong 8.314 \frac{\text{kPa}\cdot\text{ liters}}{\text{kelvin}}$). Assume that the pressure, the volume and the temperature of the gas depend all on time.

a) Suppose that one mole of ideal gas is held in a closed container with a volume of 25 liters. If the temperature of the gas is increasing at a rate of 3.5 kelvin/min, how quickly will the pressure increase?

b) Suppose instead that the temperature of the gas is held fixed at 300 kelvin, while the volume decreases at a rate of 2.0 liters/min. How quickly is the pressure of the gas increasing at the instant that the volume is 20 liters?

Ex 3. Compute the derivatives of the following functions:

a) $f(\theta) = \theta^7 + 2\theta^e - \dfrac{\pi}{\theta} + \dfrac{1}{\sqrt[2018]{\theta^{2017}}}$

b) $g(v) = \dfrac{v\ln(v)}{e^v}$

c) $w(t) = \sqrt[3]{t^2 + \cos(t)}$

d) $h(x) = \sin(x^2)e^{3x}$

e) $v(x) = \ln\left((x^3 - 5x + 1)^5\right)$

f) $f(u) = k\sqrt[k]{9e^{2\pi^2}}u$, where k is a constant

g) $u(x) = e^{\frac{1}{x^2+1}}$

h) $g(\alpha) = \tan^2(3\alpha^2 + 2)$

i) $h(t) = \cos(\beta)\sin(t)$, where β is a constant

j) $w(u) = \sin\left(\ln\left(\dfrac{u}{\cos(3u)}\right)\right)$

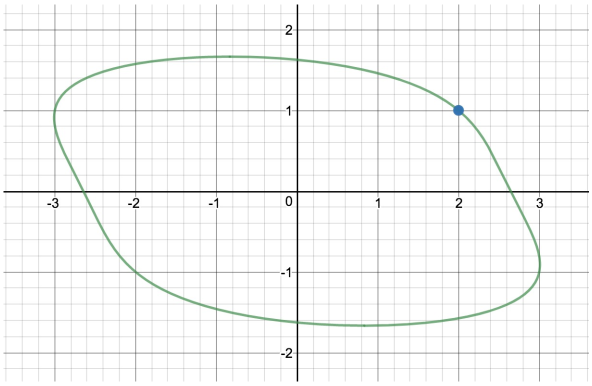
k) $g(x) = x^{\pi x}$

l) $f(t) = t^{\sin(t)+e^t}$

Ex 4. Use logarithmic differentiation to prove the power rule.

Ex 5. Consider the curve $\mathcal{C}$ given by the equation

$$y^4 + xy + x^2 = 7.$$



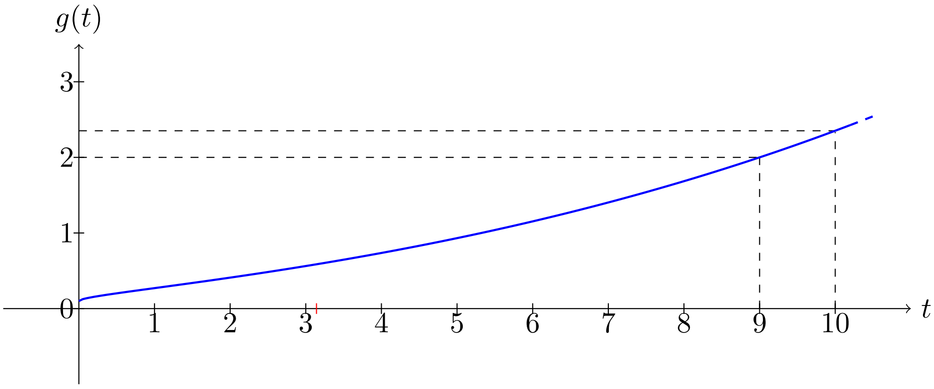
a) Use implicit differentiation to find y' $\left(\text{i.e. } \dfrac{dy}{dx}\right)$.

b) Find an equation of the tangent line to the above curve at the point $(2, 1)$.

Ex 6. An **ant** moves according to the position function:

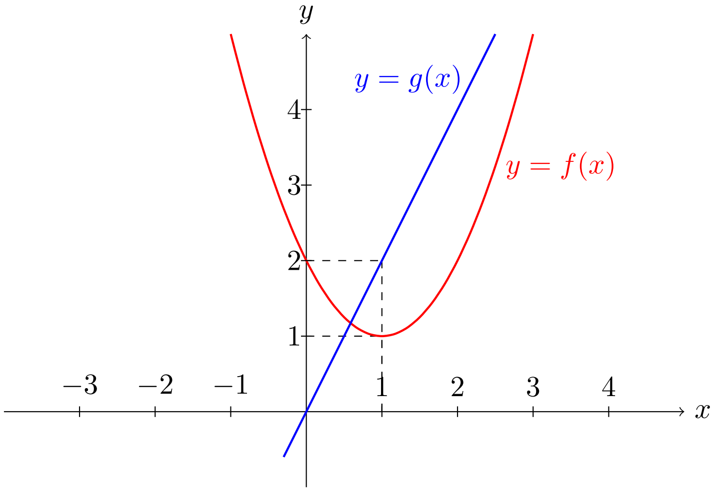
$$s(t) = 2e^{\sqrt{t}-3},$$

where t is in minutes and $s(t)$ in meters.



a) Find the linearization of $s(t)$ at $t = 9$ and use it to approximate the position of the ant at $t = 10$ min.
b) Find the velocity of the ant as a function of t.
c) Does the ant ever stop?
d) Find the acceleration of the ant as a function of t.
e) Find the acceleration at $t = 2$ min.

Ex 7.



Let f and g be the functions whose graphs are shown above and let

$$h(x) = f(x) + g(x), \qquad u(x) = f(x)g(x), \qquad v(x) = \frac{f(x)}{g(x)}, \qquad w(x) = g(f(x)).$$

Compute $h'(1)$, $u'(1)$, $v'(1)$ and $w'(1)$, without finding explicit formulas for $f(x)$ and $g(x)$.

Ex 8. **(5+5+10 points)** A couple of alligators meets at the intersection of Bruce B. Downs Blvd and Fowler Ave for organizing a romantic dinner. The male alligator starts running east at a speed of 0.4 miles per minute to chase a USF student. At the same time the female alligator starts running north at a speed of 0.3 miles per minute to chase a USF instructor. At what rate is the distance between the two alligators increasing after 5 minutes?