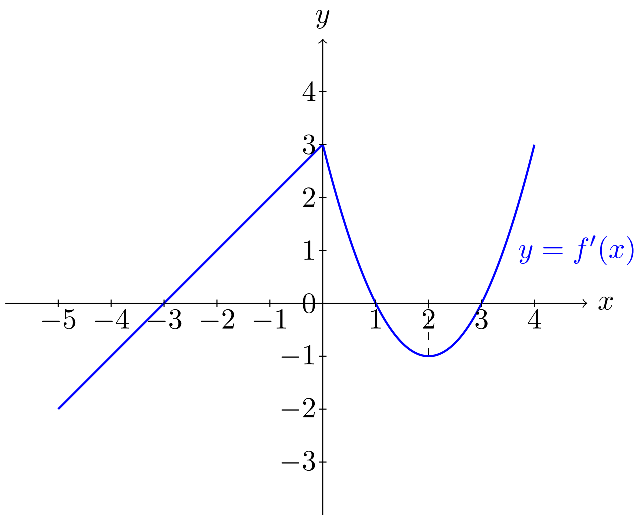
Calculus I - MAC 2311 - Section 001

Quiz - Solutions

04/04/2018

1) The graph of the derivative f' of a function f is shown below.



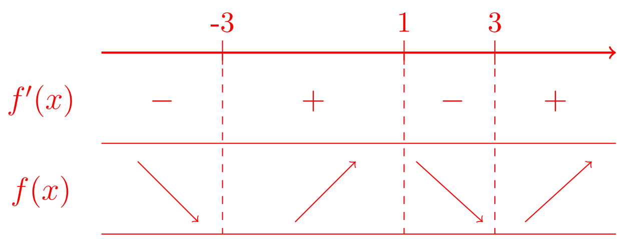
a) What are the critical numbers of f?

Since the function f' is defined everywhere (i.e. f is differentiable), then c is a critical number if and only if $f'(c) = 0$. Hence the critical numbers of f are the x-coordinates of the points at which the graph of f' crosses the x-axis:

$$\text{critical numbers}: \ x = -3, \ x = 1, \ x = 3.$$

b) Over which intervals is the function f increasing/decreasing?

We have $f'(x) > 0$ on $(-3, 1) \cup (3, \infty)$ and $f'(x) < 0$ on $(-\infty, -3) \cup (1, 3)$. Then f is increasing on $(-3, 1) \cup (3, \infty)$ and decreasing on $(-\infty, -3) \cup (1, 3)$:



c) At what numbers does f have a local minimum/maximum value?

From (b) we get that f has a local minimum value at $x = -3$ and $x = 3$, and a local maximum value at $x = 1$.

d) Over which intervals is f concave down/up?

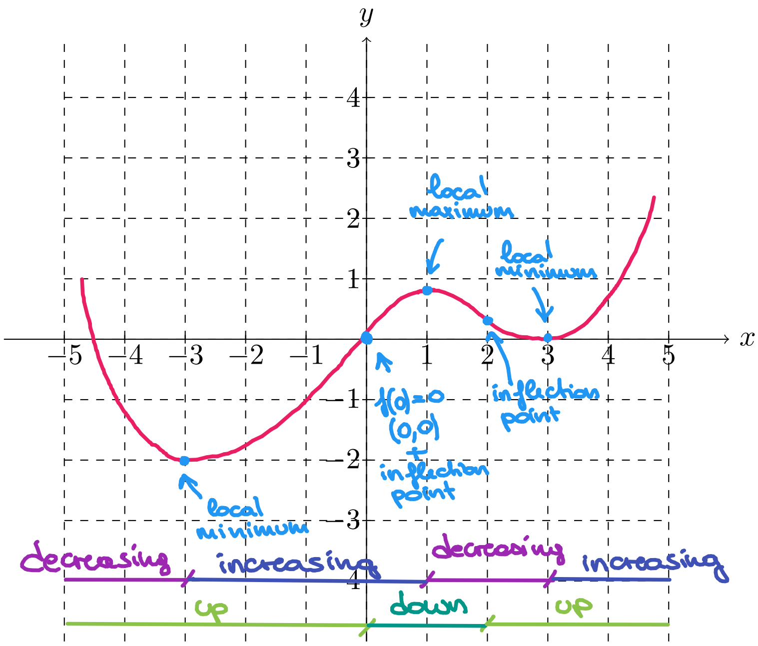
We have $f''(x) > 0$ on $(-\infty, 0) \cup (2, \infty)$ and $f''(x) < 0$ on $(0, 2)$. Then f is concave up on $(-\infty, 0) \cup (2, \infty)$ and concave down on $(0, 2)$.

		0		2	
f” (x)	$+$		$-$		$+$
$f(x)$	UP		DOWN		UP

e) What are the x-coordinates of the inflection points?

Since $f''(x)$ changes sign at $x = 0$ and at $x = 2$ and f is continuous everywhere, then $x = 0$ and $x = 2$ are the coordinates of the 2 inflection points.

e) Assuming that $f(0) = 0$, sketch a graph of f on the axis provided below.



2) [Bonus] Recall that:

Proposition: If f is a function such that $f'(x) = 0$ for all x in $\mathbb{R}$, then f is a constant function.

Use the previous result to prove that, if f and g are two differentiable functions such that $f'(x) = g'(x)$ for all x in $\mathbb{R}$, then there exists a real number c such that $f(x) = g(x) + c$.

Let us consider the function $h(x) = f(x) - g(x)$. Since $f'(x) = g'(x)$ for all x, then $h'(x) = f'(x) - g'(x) = 0$ for all x. By the proposition above, one has that $h(x)$ is a constant function, that is there exists c in $\mathbb{R}$ such that $h(x) = c$. This implies $f(x) - g(x) = c$, i.e. $f(x) = g(x) + c$.