Name and surname:

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Calculus I - MAC 2311 - Section 001

Quiz 3 - Solutions

01/31/2018

1) [2 points] State the Intermediate Value Theorem.

Theorem *(Intermediate Value Theorem). Let f be a continuous function on a closed interval $[a, b]$, with $f(a) \neq f(b)$. Then for every number N between $f(a)$ and $f(b)$ there exists c in (a, b) such that $f(c) = N$.*

2) [5 points] A residential complex near USF has a 12 feet deep swimming pool, which is currently empty. With the end of the "winter" season the management decides to fill in it again. If

$$h(t) = \frac{1}{3}t^3 - t^2 + 4t$$

represents the swimming pool water level (in feet) as a function of time (in hours), prove that between $t = 0$ hours and $t = 3$ hours there is a time at which the swimming pool is half full.

Solution:

Let us apply the Intermediate Value Theorem to our exercise in 4 steps:

♣ **Set the function and the closed interval**

Let us consider the function $h(t) = -\frac{1}{3}t^3 - t^2 + 4t$ on the closed interval $[0, 3]$.

♣ **Point out that the function is continuous on your closed interval**

The function h is continuous everywhere (and in particular on $[0, 3]$) since it is a polynomial.

♣ **Compute the value of the function at the endpoints of the interval**

We have

$$h(0) = 0 \qquad \text{and} \qquad h(3) = -\frac{1}{3} \cdot 3^3 - 3^2 + 4 \cdot 3 = 9 - 9 + 12 = 12.$$

♣ **Conclusion**

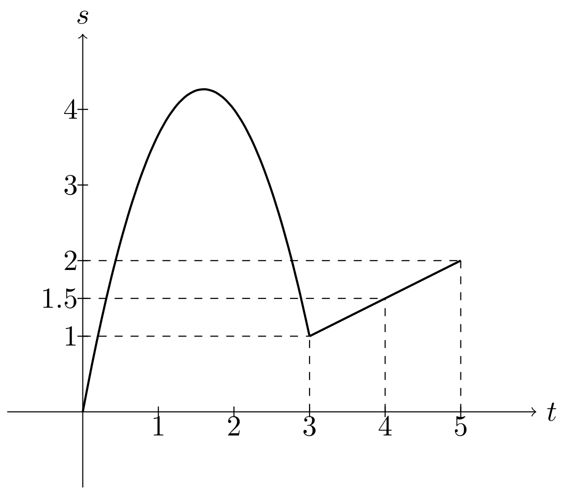
Now the swimming pool is half full when its water level it 6 feet.
Since 6 is a number between 0 and 12 ($h(0) = 0 < 6 < h(3) = 12$), then, by the Intermediate Value Theorem, there exists a time t_0 in $(0, 3)$ such that $h(t_0) = 6$, i.e. the swimming pool is half full.

3) [3 points] Compute the following limit and show all your work:

$$\lim_{x\to-\infty}\frac{-x^3-2x+3}{4x^3+5x^2+6} = \lim_{x\to-\infty}\frac{x^3\left(-\frac{x^3}{x^3}-\frac{2x}{x^3}+\frac{3}{x^3}\right)}{x^3\left(\frac{4x^3}{x^3}+\frac{5x^2}{x^3}+\frac{6}{x^3}\right)} = \lim_{x\to-\infty}\frac{x^3\left(-1-\frac{2}{x^2}+\frac{3}{x^3}\right)}{x^3\left(4+\frac{5}{x}+\frac{6}{x^3}\right)} =$$

$$= \lim_{x\to-\infty}\frac{-1-\frac{2}{x^2}+\frac{3}{x^3}}{4+\frac{5}{x}+\frac{6}{x^3}} = \text{“}\frac{-1-\frac{2}{\infty}+\frac{3}{-\infty}}{4+\frac{5}{-\infty}+\frac{6}{-\infty}}\text{”} = \frac{-1-0+0}{4+0-0} = -\frac{1}{4}.$$

4) [Bonus] Let $s(t)$ be the position function (where the position is measured in meters and the time in seconds) whose graph is the following:



What is the instantaneous velocity at $t = 4$ seconds? Why? (Do not forget the unit of measure in your answer).

Solution:

Recall that the instantaneous velocity at a time t_0 is the slope of the tangent line to the graph of the position function at the point $(t_0, s(t_0))$. When $t = 4$ seconds, the graph of the position function is a line, which is tangent to itself. Hence we have just to compute the slope of that line. We remark that the line passes through the points $(3, 1)$ and $(5, 2)$. This implies that its slope is given by:

$$\frac{2-1}{5-3} = \frac{1}{2}.$$

We get that the instantaneous velocity at $t = 4$ seconds is 0.5 m/s.