Calculus I - MAC 2311 - Section 001

Quiz 2 - Solutions

01/24/2018

1) [7.5 points] Compute the following limits. Show all your work and state any special limits used.

a) $$\lim_{x\to -3} \frac{x^2+6x+9}{x^2+2x-3} \overset{\text{plug in}}{=} \frac{(-3)^2+6(-3)+9}{(-3)^2+2(-3)-3} = \frac{9-18+9}{9-6-3} = \text{“}\frac{0}{0}\text{”}.$$

Hence we need more work for computing the limit:

$$\lim_{x\to -3} \frac{x^2+6x+9}{x^2+2x-3} = \lim_{x\to -3} \frac{(x+3)^2}{(x+3)(x-1)} = \lim_{x\to -3} \frac{x+3}{x-1} \overset{\text{plug in}}{=} \frac{-3+3}{-3-1} = \frac{0}{-4} = \mathbf{0}.$$

b) $$\lim_{t\to 2} \frac{t^2-2t}{\sqrt{2t}-2} \overset{\text{plug in}}{=} \frac{2^2-2\cdot 2}{\sqrt{2\cdot 2}-2} = \frac{4-4}{2-2} = \text{“}\frac{0}{0}\text{”}.$$

Hence we need more work for computing the limit:

$$\begin{aligned}
\lim_{t\to 2} \frac{t^2-2t}{\sqrt{2t}-2} &= \lim_{t\to 2} \frac{t^2-2t}{\sqrt{2t}-2} \cdot \frac{\sqrt{2t}+2}{\sqrt{2t}+2} = \\
&= \lim_{t\to 2} \frac{t(t-2)(\sqrt{2t}+2)}{(\sqrt{2t})^2-2^2} = \\
&= \lim_{t\to 2} \frac{t(t-2)(\sqrt{2t}+2)}{2t-4} = \\
&= \lim_{t\to 2} \frac{t(t-2)(\sqrt{2t}+2)}{2(t-2)} = \\
&= \lim_{t\to 2} \frac{t(\sqrt{2t}+2)}{2} \overset{\text{plug in}}{=} \frac{2\cdot(\sqrt{2\cdot 2}+2)}{2} = \frac{8}{2} = \mathbf{4}.
\end{aligned}$$

c) $$\lim_{\theta\to 0} \frac{\sin(2018\theta)}{\theta} \overset{\text{plug in}}{=} \frac{\sin(2018\cdot 0)}{0} = \text{“}\frac{0}{0}\text{”}.$$

Hence we need more work for computing the limit:

$$\begin{aligned}
\lim_{\theta\to 0} \frac{\sin(2018\theta)}{\theta} &= \lim_{\theta\to 0} \frac{\sin(2018\theta)}{\theta} \cdot \frac{2018}{2018} = \\
&= \lim_{\theta\to 0} 2018 \cdot \frac{\sin(2018\theta)}{2018\theta} = \\
&= 2018 \cdot \lim_{\theta\to 0} \frac{\sin(2018\theta)}{2018\theta} \overset{\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1}{=} 2018\cdot 1 = \mathbf{2018}.
\end{aligned}$$

2) [2.5 points] Give the definition of a function which is continuous at a number a.

A function is *continuous* at a number a if $\lim_{x\to a} f(x) = f(a)$.

3) [Bonus] A student says:

The function

$$f(x) = \begin{cases} \cos(\pi x), & \text{when } x \leq 1 \\ -\sin\left(\frac{\pi}{2}x\right), & \text{when } x > 1 \end{cases}$$

is discontinuous at $x = 1$ because $x = 1$ is a "breaking point" for f.

Do you agree or disagree with the student? Explain your answer.

Solution

I stongly disagree with the student. A piecewise function can also be continuous at its breaking point. Indeed in this case we have:

- $\lim_{x\to 1^-} f(x) \overset{x\leq 1}{=} \lim_{x\to 1^-} \cos(\pi x) = \cos(\pi \cdot 1) = \cos(\pi) = -1;$
- $\lim_{x\to 1^+} f(x) \overset{x\geq 1}{=} \lim_{x\to 1^+} -\sin\left(\frac{\pi}{2}x\right) = -\sin\left(\frac{\pi}{2}\cdot 1\right) = -\sin\left(\frac{\pi}{2}\right) = -1;$
- $f(1) \overset{x=1}{=} \cos(\pi \cdot 1) = \cos(\pi) = -1.$

Since $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$, which is equivalent to $\lim_{x\to 1} f(x) = f(1)$, the function f is continuous at $x = 1$ even if this is a "breaking point". This is very clear if we look at the graph of $f(x)$.