Calculus I - MAC 2311 - Section 001
Homework 3

Instructions: Solve the following exercises in a **separate sheet of paper**. Be tidy and organized! You can work on the exercises with your friends (or enemies!) but the final editing has to be yours. The homework has to be returned **by Wednesday April 4, 11 am**. The total number for this homework is 110 (there are 10 extra points). The grade you will receive for this homework will count as a part of *Quizzes and handwritten homework* component of the total grade (15%).

Ex 1. (25 points) Compute the following limits. If you use l'Hospital's Rule state which type of indeterminate form you have.

a) $\displaystyle\lim_{x\to 1} \frac{\cos(\pi x) + e^{x-1}}{x-1}$

b) $\displaystyle\lim_{x\to -\infty} \frac{\tan\left(\frac{1}{x}\right) + 1}{\arctan(x)}$

c) $\displaystyle\lim_{x\to \infty} (e^x + 1)^{e^{-2x}}$

d) $\displaystyle\lim_{x\to \infty} \frac{\pi - 2\arctan(x)}{e^{-x}}$

e) $\displaystyle\lim_{x\to 0^+} x \cdot \ln\left(\ln(x+1)\right)$

Ex 2. (25 points) Consider the function

$$f(x) = \frac{1}{x} \cdot e^x.$$

a) Find the domain of definition of f.

b) Find the horizontal and vertical asymptotes.

c) Find the critical numbers of f.

d) Find the intervals over which f is increasing/decreasing and the local maximum/minimum value of f.

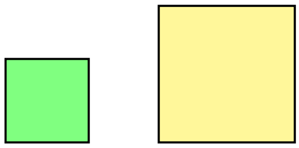
e) After having shown that

$$f''(x) = \frac{e^x(x^2 - 2x + 2)}{x^3},$$

find the intervals where f is concave upward/downward and the inflection points of f, if any. (*Hint: note that the equation $x^2 - 2x + 2 = 0$ has no real solutions.)*

f) Sketch the graph of $y = f(x)$, by using the information you collected above.

Ex 3. **(20 points)** A farmer has 400 feet of fencing and wants to fence two square fields, each one on all four sides (see the picture below). What are the length of the sides of the two square fields when they cover (together) the least area?



Ex 4. **(20 points)** A calculus student wakes up late for his calculus test and starts driving to school, trying to make it on time. His home is 10 miles away from USF and the speed limit on all the roads on his way is 35 miles per hour.

a) A webcam located 2 miles away from his home records the car of the student at 9:18 am, and another one located 6 miles away from his home records it at 9:24 am. Prove that the student will be fined for speeding.

b) After 9:24 am the student complies with the speed limit. Show that he will not arrive on time for the test at 9:30 am ☹.

Hint: You can consider the position function $f(t)$, where $f(t)$ represents at a time t the distance of the car of the student from his house. Be careful with the units of measure and recall that 1 min $= \frac{1}{60}$ hour.

Conclusion: Respect always the speed limits and set more than one alarm for the day of your test! ☺.

Ex 5. **(20 points)** Which statements are True/False? Justify your answers.

a) The function $f(x) = \ln(x+1)$ has an absolute maximum and minimum value on $[-1, 1]$.

b) If f is a function such that $f''(x) > 0$ for all x, and $f'(1) = 1$, then the absolute maximum value of f on the interval $[1, 3]$ is $f(3)$.

c) The function

$$f(x) = \begin{cases} -x^2, & \text{when } x < 0 \\ x^2 + 1, & \text{when } x \geq 0 \end{cases}$$

has an inflection point at $(0, 1)$ since $f''(x) < 0$ on $(-\infty, 0)$ and $f''(x) > 0$ on $(0, \infty)$.

d) Let f be a function such that $f'(x) \neq 0$ for all x. Then the equation $f(x) = 0$ can have two different solutions x_1 and x_2.