Calculus I - MAC 2311 - Section 001

Homework 2

Instructions: Solve the following exercises in a **separate sheet of paper**. Be tidy and organized! You can work on the exercises with your friends (or enemies!) but the final editing has to be yours. The homework has to be returned **by Wednesday February 28, 11 am**. The total number for this homework is 104 (there are 4 extra points). The grade you will receive for this homework will count as a part of *Quizzes and handwritten homework* component of the total grade (15%).

Ex 1. (24 points) Differentiate with respect to the indicated variable. If k appears in the function, treat it as a constant. Before starting computing your derivative, think if it is possible to simplify the function. Show all your work.

a) $\frac{d}{dx}\left[2x^7 - 3x^5 - 5x^3 + 7x^2\right]$

b) $\frac{d}{du}\left[-\frac{u^3}{u^{11}} + \frac{\sqrt[3]{u^2}}{\sqrt[5]{u^6}}\right]$

c) $\frac{d}{dx}\left[\frac{x^3 + 5x\sin(x)}{x}\right]$

d) $\frac{d}{dt}\left[t^2\cos(t)\right]$

e) $\frac{d}{dx}\left[\frac{e^{\sqrt{2018}}}{\pi^3}\right]$

f) $\frac{d}{dx}\left[\frac{x^2}{\ln(x)}\right]$

g) $\frac{d}{d\theta}\left[\sqrt{\cos(\theta)+1}\right]$

h) $\frac{d}{d\theta}\left[k\cos\left(\sqrt{\theta}\right)+1\right]$

i) $\frac{d}{dx}\left[e^{2\cos(x)}\right]$

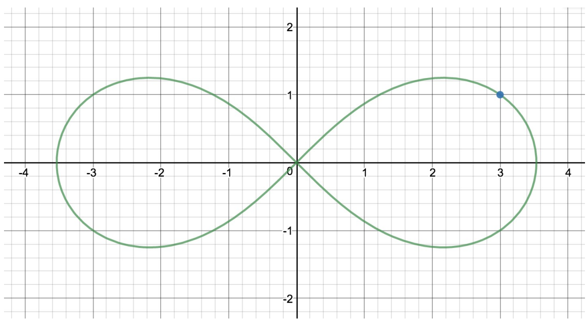
j) $\frac{d}{d\alpha}\left[\tan(\sin(\pi\alpha))\right]$

k) $\frac{d}{dx}\left[e^{\ln(\tan(x))}\right]$

l) $\frac{d}{dt}\left[\ln\left(e^{\sqrt{k}e^t}\right)\right]$

Ex 2. (10+10 points) Consider the **lemniscate** curve given by the following equation:

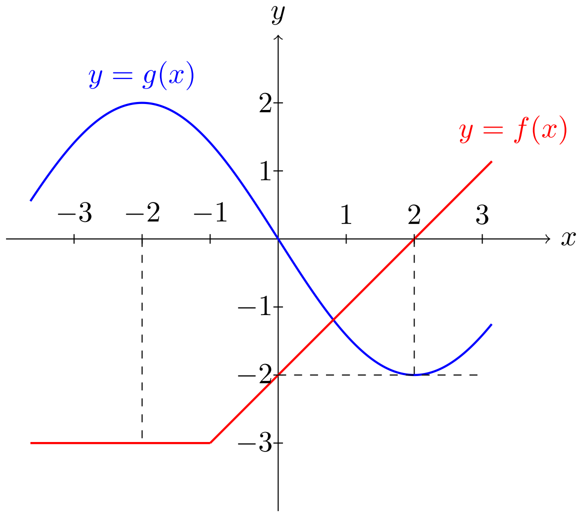
$$2x^4 + 4x^2y^2 + 2y^4 = 25x^2 - 25y^2.$$



a) Use implicit differentiation to find y' (i.e. $\frac{dy}{dx}$).

b) Find an equation of the tangent line to the above curve at the point $(3, 1)$.

Ex 3. (5+5+5+5 points)



Let f and g be the functions whose graphs are shown above and let

$$h(x) = f(x) + g(x), \qquad u(x) = f(x)g(x), \qquad v(x) = \frac{f(x)}{g(x)}, \qquad w(x) = f(g(x)).$$

Compute $h'(2)$, $u'(2)$, $v'(2)$ and $w'(2)$, without finding explicit formula for $f(x)$ and $g(x)$.

Ex 4. (20 points) It is the Sunday before the second test. The calculus student of HW1, who was disappointed by his experience on Floridian mountains, decides this time to have a productive study break at Clearwater beach. After filling a bucket with dry sand, he starts pouring the sand on the ground at a steady rate of 5 cm^3/s. He notices that, at each time, the sand forms a conical pile whose height is always equal to half of the diameter of its base. How fast is the radius of the conical pile increasing when the height is 10 cm?

(You can find a suitable formula for the volume in the "Geometry" section of Reference Page 1 at the end of your textbook.)

Ex 5. (5+5+5+5 points) Which statements are True/False? Justify your answers.

a) If $f(x) = x^{\tan(x)}$ then $f'(x) = \tan(x) \cdot x^{\tan(x)-1}$.

b) If $f(x) = \sin(x)$ then $f'''(0) = 0$.

c) If the graph of a physical quantity F as function of time is a line, then the rate of change of F with respect to time is constant.

d) If $f(t) = e^{x+2}$, then $f^{-1}(e^2) = \{0\}$.