Calculus I - MAC 2311 - Section 003

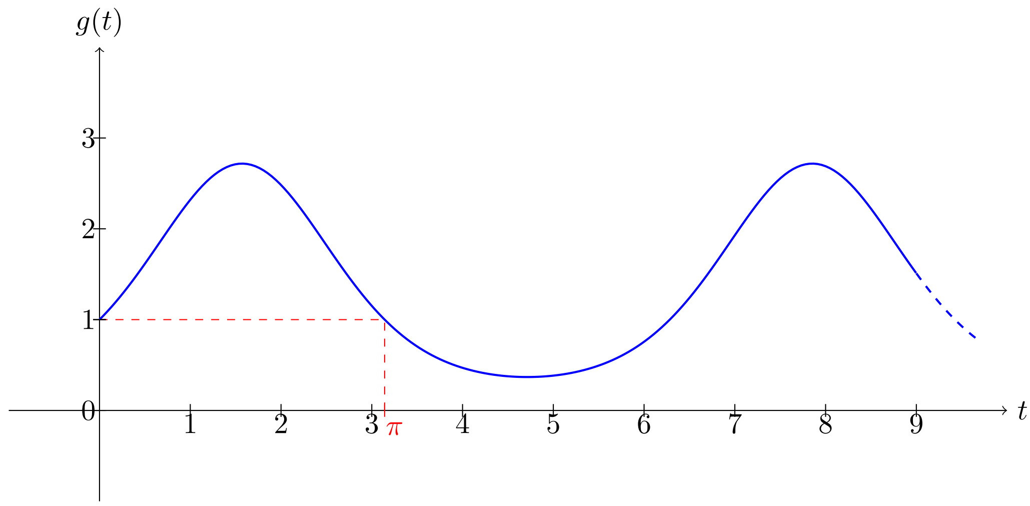
Review session Test 2

10/11/2018

Ex 1. The equation of motion of a **goldfish** which swims horizontally in a bowl is:

$$g(t) = e^{\sin(t)},$$

where t is in seconds and $g(t)$ is in inches.



a) Find the linearization at $t = \pi$ and use it to approximate the position of the goldfish at $t = 3$ sec.
b) Find the velocity of the goldfish as a function of t.
c) When is the velocity zero?
d) Find the acceleration as a function of t.
e) Find the acceleration at $t = \pi$.

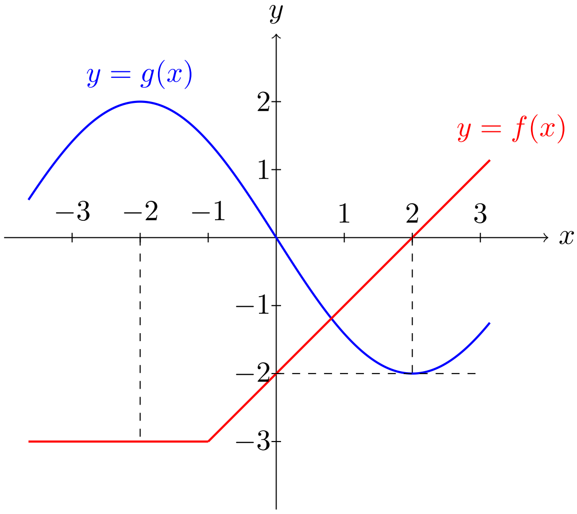
Ex 2. Prove that $\dfrac{d}{dt}[\sin(t)\tan(t)] = \sin(t) + \tan(t)\sec(t)$.

Ex 3. Compute the derivative of the following functions. Before starting computing your derivative, think if it is possible to simplify the function.

a) $f(x) = x^{x^2+2x}$
b) $f(u) = e^{u^2} \cdot \ln u$
c) $g(x) = \ln\left((\pi\sqrt{x})^e\right)$
d) $h(s) = \dfrac{1}{e^{\sin(2ks)}}$, where k is a constant.
e) $w(\theta) = e^{\ln(\ln(\theta^2))}$
f) $f(t) = (\sin(t))^{2t}$

Ex 4. Use logarithmic differentiation to prove the power rule.

Ex 5.



Let f and g be the functions whose graphs are shown above and let

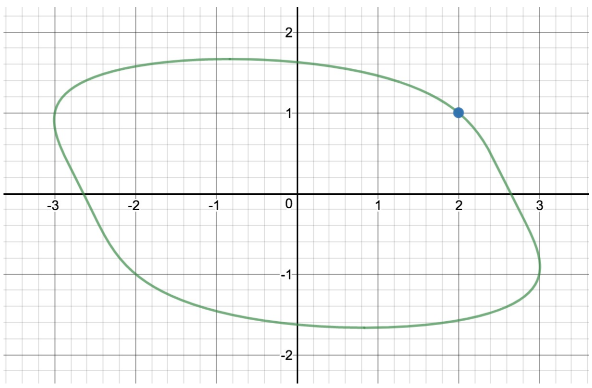
$$h(x) = f(x) + g(x), \qquad u(x) = f(x)g(x), \qquad v(x) = \frac{f(x)}{g(x)}, \qquad w(x) = f(g(x)).$$

Compute $h'(2)$, $u'(2)$, $v'(2)$ and $w'(2)$, without finding explicit formula for $f(x)$ and $g(x)$.

Ex 6. It is the Sunday before the second test. A calculus student decides to have a productive study break at Clearwater beach. After filling a bucket with dry sand, he starts pouring the sand on the ground at a steady rate of 5 cm^3/s. He notices that, at each time, the sand forms a conical pile whose height is always equal to half of the diameter of its base. How fast is the radius of the conical pile increasing when the height is 10 cm?

Ex 7. Consider the curve $\mathcal{C}$ given by the equation

$$y^4 + xy + x^2 = 7.$$



a) Use implicit differentiation to find y' $\left(\text{i.e. } \dfrac{dy}{dx}\right)$.

b) Find an equation of the tangent line to the above curve at the point $(2, 1)$.

Ex 8. Compute the derivatives of the following functions:

a) $f(\theta) = \theta^7 + 2\theta^e - \dfrac{\pi}{\theta} + \dfrac{1}{\sqrt[2018]{\theta^{2017}}}$

b) $g(v) = \dfrac{v\ln(v)}{e^v}$

c) $w(t) = \sqrt[3]{t^2 + \cos(t)}$

d) $h(x) = \sin(x^2)e^{3x}$

e) $v(x) = \ln\left((x^3 - 5x + 1)^5\right)$

f) $f(u) = k\sqrt[k]{9e^{2\pi^2}}u$, where k is a constant

g) $u(x) = e^{\frac{1}{x^2+1}}$

h) $g(\alpha) = \tan^2(3\alpha^2 + 2)$

i) $h(t) = \cos(\beta)\sin(t)$, where β is a constant

j) $w(u) = \sin\left(\ln\left(\dfrac{u}{\cos(3u)}\right)\right)$

k) $g(x) = x^{\pi x}$

l) $f(t) = t^{\sin(t)+e^t}$