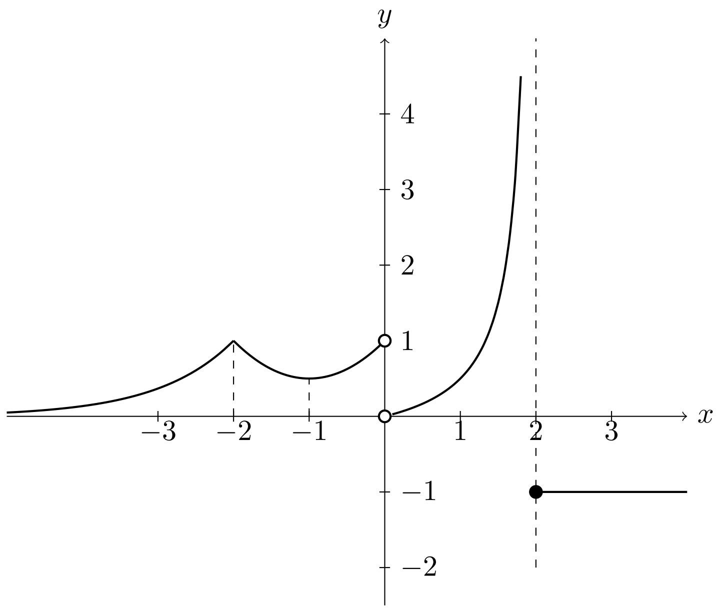
Calculus I - MAC 2311 - Section 003

Review session - Test 1

09/13/2018

Ex 1. The graph of a function f is given.



a) Find the quantities: $\lim_{x\to-\infty} f(x)$, $\lim_{x\to-2} f(x)$, $\lim_{x\to 0} f(x)$, $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to\infty} f(x)$.

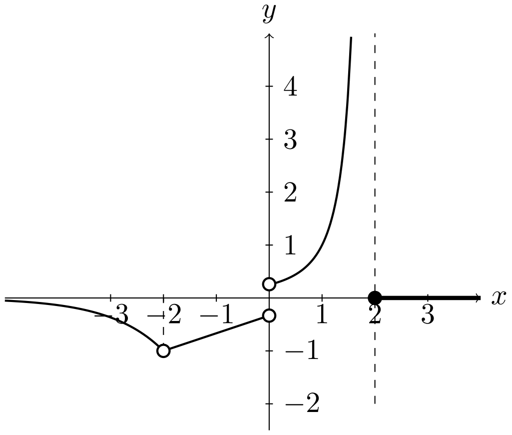
b) Write the equations of the horizontal asymptotes and vertical asymptotes of the function f, if any. Explain your answer in terms of limits.

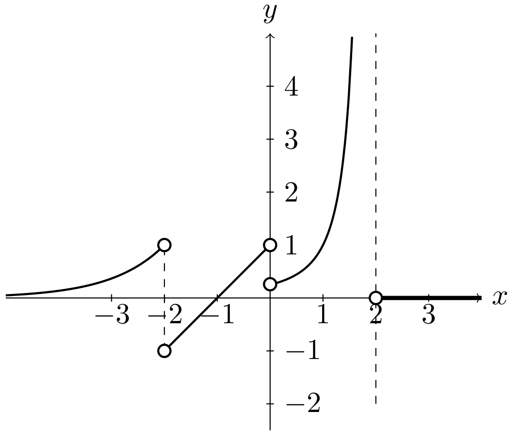
c) Over which intervals is $f(x)$ continuous? For each discontinuity, state its kind and explain your answer.

d) For which numbers x do we have $f'(x) = 0$? Why?

e) For which numbers x is the function $f(x)$ <u>not</u> differentiable? Why?

f) Which one of the following graphs may be the graph of the *derivative* of $f(x)$? Why?





Ex 2. Sketch the graph of a function f which satisfies **all** the following conditions:

a) $\lim_{x\to-\infty} f(x) = 2$,

b) $f(-2) = 3$,

c) $\lim_{x\to1^-} f(x) = -\infty$,

d) $f(1) = 0$

e) $\lim_{x\to1^+} f(x) = 0$,

f) $\lim_{x\to\infty} f(x) = -1$,

Ex 3. Sketch the graph of a function f which satisfies **all** the following conditions:

a) $y = -1$ is a horizontal asymptote,

b) $x = -2$ is a remouvable discontinuity,

c) $x = 0$ is a vertical asymptote,

d) $f(0) = 1$,

e) $\lim_{x\to0^+} f(x) = 1$,

f) f is not differentiable at $x = 2$,

g) $f'(x)$ is constant for all x in $(2, \infty)$.

Ex 4. An alligator moves according to the position function $s(t) = t^2 - 4t - 1$, where position is measured in meters and time in seconds.

a) Prove that between 0 and 5 seconds there is a time t_0 at which $s(t_0) = 0$.
b) Find the instantaneous velocity $v(t)$ at each time t, by using the definition of derivative. (Recall that $v(t) = s'(t)$).
c) What is the velocity of the alligator at $t = 5$ seconds?
d) At what time(s) is the velocity of the alligator zero?

Ex 5. Let f be the piecewise function defined as:

$$f(x) = \begin{cases} \frac{x^3-2cx-2}{x}, & \text{when } x < -1; \\ -c^2 \cdot \cos(-\pi x), & \text{when } x \geq -1. \end{cases}$$

a) Is $f(x)$ continuous on $(-\infty, -1)$? Why?
b) Is $f(x)$ continuous on $(-1, \infty)$? Why?
c) Define what it means for f to be continuous at $x = -1$.
d) Find the value(s) of c that make the function continuous everywhere.

Ex 6. Consider the rational function:

$$f(x) = \frac{-2x^2 + 2x + 12}{x^2 + 3x + 2}.$$

a) Find the domain of $f(x)$.
b) Write the equation(s) of the vertical asymptote(s) of $f(x)$ and justify your answer.
c) Compute $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to\infty} f(x)$.
d) Write the equation(s) of the horizontal asymptote(s) of $f(x)$ and justify your answer.

Ex 7. Find the derivative of the function $f(x) = \sqrt{x} + x$. Then, write the equation of the tangent line to the curve $y = f(x)$ at the point $P(4, 6)$.

Ex 8. Let $f(x)$ be a function such that:

$$\frac{\sin(x)}{3x} \leq f(x) \leq \frac{3x^3 + x}{3x}, \quad \text{for all } x \neq 0.$$

Compute $\lim_{x\to 0} f(x)$, and name any theorem you used.

Ex 9. Compute the following limits:

a) $\displaystyle\lim_{x\to 0} \frac{x}{x^2+1}$

b) $\displaystyle\lim_{x\to -7} \frac{x+7}{x^2+6x-7}$

c) $\displaystyle\lim_{t\to 3} \frac{\sqrt{3t}-3}{t^2-3t}$

d) $\displaystyle\lim_{x\to 0} \frac{x}{\sqrt{2+x}-\sqrt{2-x}}$

e) $\displaystyle\lim_{\theta\to 0} \frac{3\sin(12\theta)}{4\theta}$

f) $\displaystyle\lim_{x\to \infty} \frac{2x^5-x^3+3}{6x^5+1}$

g) $\displaystyle\lim_{x\to -\infty} \frac{x^3-x^2+x-1}{1-x}$

h) $\displaystyle\lim_{t\to \infty} \frac{t+1}{t^2+1}$

i) $\displaystyle\lim_{x\to 2} \frac{x-3}{(x-2)^2}$

j) $\displaystyle\lim_{x\to 0^+} \frac{\sin(x+\frac{\pi}{2})+1}{x}$

k) $\displaystyle\lim_{x\to 1^-} \frac{-|x-1|}{x-1}$

l) $\displaystyle\lim_{\theta\to 0} \frac{\sin(2018\theta)}{\sin(2019\theta)}$

Ex 10. Match the graph of each function in (a)-(d) with the graph of its derivative in I-IV. Give reasons for your choice.