# Calculus I - MAC 2311 - Section 003

## Quiz 7 - Solutions

11/14/2018

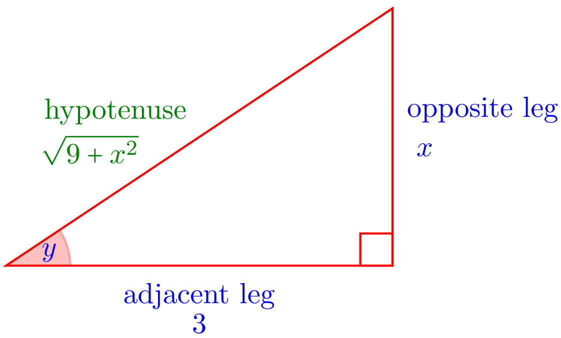
**Instructions:** The total number of points of this quiz is 11, but your grade will be the minimum between your score and 10. You will get an extra point if you solve correctly the last exercise. **Calculators are not allowed** for this quiz.

1) Showing all your work, simplify the expression $\cos\left(\tan^{-1}\left(\frac{x}{3}\right)\right)$.

*Solution:*

Let us set $y = \tan^{-1}\left(\frac{x}{3}\right)$. Then $\tan(y) = \frac{x}{3}$ with $-\frac{\pi}{2} < y < \frac{\pi}{2}$.

We recall that in a right triangle $\tan(y) = \frac{\text{opposite leg}}{\text{adjacent leg}}$. Here $\tan(y) = \frac{x}{3}$, hence we can consider the right triangle with opposite leg of length $x$ and adjacent leg of length 3 (see the picture below):




Then:

$$\cos\left(\tan^{-1}\left(\frac{x}{3}\right)\right) = \cos(y) = \frac{\text{adjacent leg}}{\text{hypothenuse}} = \frac{3}{\sqrt{9+x^2}}.$$

2) Compute the derivatives of the following functions:

a) $f(x) = (3x+1)\cdot\arctan(x^3)$.

*Solution:*

Let $f(x) = (3x+1)\cdot\arctan(x^3)$. We have:

$$\begin{aligned}
f'(x) &= \left((3x+1)\cdot\arctan(x^3)\right)' = \\
&= (3x+1)'\cdot\arctan(x^3) + (3x+1)\cdot\left(\arctan(x^3)\right)' = \\
&= 3\arctan(x^3) + (3x+1)\cdot\frac{1}{1+x^6}\cdot\left(x^3\right)' = \\
&= 3\arctan(x^3) + (3x+1)\cdot\frac{1}{1+x^6}\cdot 3x^2 = \\
&= 3\arctan(x^3) + \frac{9x^3+3x^2}{1+x^6}.
\end{aligned}$$

b) $g(t) = \dfrac{\ln(2t)}{\arccos(t)}$

*Solution:*

Let $g(t) = \dfrac{\ln(2t)}{\arccos(t)}$. We have:

$$\begin{aligned} g'(t) &= \left(\frac{\ln(2t)}{\arccos(t)}\right)' = \\ &= \frac{(\ln(2t))' \cdot \arccos(t) - \ln(2t) \cdot (\arccos(t))'}{(\arccos(t))^2} = \\ &= \frac{\frac{2}{2t} \cdot \arccos(t) - \ln(2t) \cdot \left(-\frac{1}{\sqrt{1-t^2}}\right)}{(\arccos(t))^2} = \\ &= \frac{\frac{\arccos(t)}{t} + \frac{\ln(2t)}{\sqrt{1-t^2}}}{(\arccos(t))^2}. \end{aligned}$$

3) Prove that $\frac{d}{dx}[\arccos(x)] = -\dfrac{1}{\sqrt{1-x^2}}$.

*Solution:*

We set $y = \arccos(x)$. Then $\cos y = x$ with $0 \le y \le \pi$. We want to prove that $\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$. We have:

$$\begin{gathered} \frac{d}{dx}(\cos y) = \frac{d}{dx}(x) \\ \Downarrow \\ -\sin y \cdot \frac{dy}{dx} = 1 \\ \Downarrow \\ \frac{dy}{dx} = -\frac{1}{\sin y} \end{gathered}$$

Now, from the Pythagorean identity $\cos^2(y) + \sin^2(y) = 1$, we obtain:

$$\begin{gathered} \sin^2(y) = 1 - \cos^2(y) \\ \Downarrow \sin y \ge 0 \text{ when } y \in [0, \pi] \\ \sin y = \sqrt{1 - \cos^2(y)} \\ \Downarrow \cos y = x \\ \sin y = \sqrt{1 - x^2}, \end{gathered}$$

and therefore $\dfrac{dy}{dx} = -\dfrac{1}{\sin y} = -\dfrac{1}{\sqrt{1-x^2}}$.

4) Compute $\arcsin(\sin(\pi))$ and show all your work.

*Solution:*

We have:

$$\arcsin\left(\sin(\pi)\right) \overset{\sin(\pi)=0}{=} \arcsin\left(0\right) = 0.$$

Note that, since $\pi \notin \left[\frac{\pi}{2}, \frac{\pi}{2}\right]$, we can not apply the cancellation equation

$$\arcsin\left(\sin(x)\right) = x, \text{ for all } x \in \left[\frac{\pi}{2}, \frac{\pi}{2}\right].$$