Calculus I - MAC 2311 - Section 003

Quiz 3 - Solutions

09/26/2018

1) For each of the following functions compute its derivative:

a) $f(x) = x^{10} - \frac{3x^5}{5} - \frac{3}{x^3} + \sqrt[4]{x^3}$

Solution:

$$\begin{aligned} f'(x) &= \left(x^{10} - \frac{3x^5}{5} - \frac{3}{x^3} + \sqrt[4]{x^3}\right)' = \\ &= (x^{10})' - \left(\frac{3}{5}x^5\right)' - (3x^{-3})' + \left(x^{\frac{3}{4}}\right)' = \\ &= (x^{10})' - \frac{3}{5}(x^5)' - 3\left(x^{-3}\right)' + \left(x^{\frac{3}{4}}\right)' = \\ &= 10x^9 - \frac{3}{5} \cdot 5x^4 - 3 \cdot (-3) \cdot x^{-4} + \frac{3}{4} \cdot x^{\frac{3}{4}-1} = \\ &= 10x^9 - 3x^4 + \frac{9}{x^4} + \frac{3}{4\sqrt[4]{x}}. \end{aligned}$$

b) $\frac{d}{dt}\left[t^5 \sin(t)\right] =$

Solution:

$$\begin{aligned} \frac{d}{dt}\left[t^5 \sin(t)\right] &= \frac{d}{dt}[t^5] \cdot \sin(t) + t^5 \cdot \frac{d}{dt}[\sin(t)] = \\ &= 5t^4 \cdot \sin(t) + t^5 \cdot \cos(t). \end{aligned}$$

c) $f(\theta) = \tan\left(2\cos(\theta) + \sqrt{\theta}\right).$

Solution:

$$\begin{aligned} f'(\theta) &= \left[\tan\left(2\cos(\theta) + \sqrt{\theta}\right)\right]' = \\ &= \sec^2\left(2\cos(\theta) + \sqrt{\theta}\right) \cdot \left(2\cos(\theta) + \sqrt{\theta}\right)' = \\ &= \sec^2\left(2\cos(\theta) + \sqrt{\theta}\right) \cdot \left(-2\sin(\theta) + \frac{1}{2\sqrt{\theta}}\right). \end{aligned}$$

d) $f(u) = \dfrac{u + 1 + \sin(7u)}{u^2}$.

Solution:

$$f'(u) = \left(\frac{u + 1 + \sin(7u)}{u^2}\right)' =$$
$$= \frac{(u + 1 + \sin(7u))' \cdot u^2 - (u + 1 + \sin(7u))(u^2)'}{u^4} =$$
$$= \frac{(1 + 0 + \cos(7u) \cdot 7) \cdot u^2 - (u + 1 + \sin(7u)) \cdot 2u}{u^4}.$$

e) $f(x) = \left(\sin\left(\sqrt[3]{x}\right)\right)^2$.

Solution:

$$f'(x) = \left[\left(\sin\left(\sqrt[3]{x}\right)\right)^2\right]' =$$
$$= 2\left(\sin\left(\sqrt[3]{x}\right)\right) \cdot \left(\sin\left(\sqrt[3]{x}\right)\right)' =$$
$$= 2\left(\sin\left(\sqrt[3]{x}\right)\right) \cdot \cos\left(\sqrt[3]{x}\right) \cdot \left(\sqrt[3]{x}\right)' =$$
$$= 2\left(\sin\left(\sqrt[3]{x}\right)\right) \cdot \cos\left(\sqrt[3]{x}\right) \cdot \frac{1}{3\sqrt[3]{x^2}}.$$

2) Consider the following piecewise defined function:

$$f(x) = \begin{cases} \dfrac{1}{x+2} & \text{if } x < -3 \\ x^2 + 3x - 1 & \text{if } x \geq -3 \end{cases}$$

Is f continuous at $x = -3$? Justify your answer.

We notice that $x = -3$ is the "breaking point" for our piecewise-defined function. Then, we have to compute $\lim_{x \to -3^-} f(x)$, $\lim_{x \to -3^+} f(x)$ and $f(-3)$:

- $\displaystyle\lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} \frac{1}{x+2} = \frac{1}{-3+2} = -1$
- $\displaystyle\lim_{x \to -3^+} f(x) = \lim_{x \to -3^+} x^2 + 3x - 1 = (-3)^2 + 3(-3) - 1 = 9 - 9 - 1 = -1$
- $f(-3) = (-3)^2 + 3(-3) - 1 = -1$.

Since $\displaystyle\lim_{x \to -3^-} f(x) = \lim_{x \to -3^+} f(x) = f(-3)$, then f is continuous at $x = -3$.

3) Compute the following derivative:

$$\frac{d}{dx}\left[k\cos(kx)+k\right],$$

where k is a constant.

Solution:

Since k is a constant, we will treat it like a number. Then:

$$\begin{aligned}\frac{d}{dx}\left[k\cos(kx)+k\right] &= \frac{d}{dx}[k\cos(kx)]+\frac{d}{dx}[k] = \\ &= k\frac{d}{dx}[\cos(kx)]+0 = \\ &= k\left(-\sin(kx)\right)\cdot k = \\ &= -k^2\sin(kx).\end{aligned}$$