Calculus I - MAC 2311 - Section 003

Quiz 2 - Solutions

09/05/2018

1) [7.5 points] Compute the following limits. Show all your work and state any special limits used.

a) $$\lim_{x\to 4} \frac{x^2-5x+4}{x^2-2x-8} \overset{\text{plug in}}{=} \frac{(4)^2-5\cdot 4+4}{(4)^2-2\cdot 4-8} = \frac{16-20+4}{16-8-8} = \text{“}\frac{0}{0}\text{”}.$$

Hence we need more work for computing the limit:

$$\lim_{x\to 4} \frac{x^2-5x+4}{x^2-2x-8} = \lim_{x\to 4} \frac{(x-4)(x-1)}{(x-4)(x+2)} = \lim_{x\to 4} \frac{x-1}{x+2} \overset{\text{plug in}}{=} \frac{4-1}{4+2} = \frac{3}{6} = \mathbf{\frac{1}{2}}.$$

b) $$\lim_{t\to 1} \frac{1-t^2}{\sqrt{t}-1} \overset{\text{plug in}}{=} \frac{1-(1)^2}{\sqrt{1}-1} = \text{“}\frac{0}{0}\text{”}.$$

Hence we need more work for computing the limit:

$$\begin{aligned}\lim_{t\to 1} \frac{1-t^2}{\sqrt{t}-1} &= \lim_{t\to 1} \frac{1-t^2}{\sqrt{t}-1}\cdot\frac{\sqrt{t}+1}{\sqrt{t}+1} = \\ &= \lim_{t\to 1} \frac{(1-t^2)(\sqrt{t}+1)}{(\sqrt{t})^2-1} = \\ &= \lim_{t\to 1} \frac{(1-t)(1+t)(\sqrt{t}+1)}{t-1} = \\ &= \lim_{t\to 1} \frac{-(t-1)(1+t)(\sqrt{t}+1)}{t-1} = \\ &= \lim_{t\to 1} \frac{-(1+t)(\sqrt{t}+1)}{1} \overset{\text{plug in}}{=} \frac{-2\cdot 2}{1} = \mathbf{-4}.\end{aligned}$$

c) $$\lim_{\theta\to 0} \frac{\sin(5\theta)}{10\theta} \overset{\text{plug in}}{=} \frac{\sin(5\cdot 0)}{10\cdot 0} = \text{“}\frac{0}{0}\text{”}.$$

Hence we need more work for computing the limit:

$$\begin{aligned}\lim_{\theta\to 0} \frac{\sin(5\theta)}{10\theta} &= \lim_{\theta\to 0} \frac{1}{2}\cdot\frac{\sin(5\theta)}{5\theta} = \\ &= \frac{1}{2}\cdot\lim_{\theta\to 0}\frac{\sin(5\theta)}{5\theta} \overset{\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1}{=} \frac{1}{2}\cdot 1 = \mathbf{\frac{1}{2}}.\end{aligned}$$

2) [2.5 points] State the *Squeeze theorem.*

Let f, g, h be functions defined near a (except possibly at a). Suppose that:

1) $g(x) \leq f(x) \leq h(x)$ for all x near a (except possibly at a);
2) $\lim_{x\to a} g(x) = \lim_{x\to a} h(x) = L$.

Then:

$$\lim_{x\to a} f(x) = L$$

3) [1 point] Let $f(x)$ be a function such that $-1 \leq f(x) \leq x^2 - 2x$, for all x. Compute $\lim_{x\to 1} f(x)$.

Solution

Let $g(x) = -1$ and $h(x) = x^2 - 2x$. We have:

1) $g(x) \leq f(x) \leq h(x)$, for all x (so, in particular near 1);
2) $\lim_{x\to 1} g(x) = \lim_{x\to 1} h(x) = -1.$.

Then, by the *Squeeze Theorem*, we get $\lim_{x\to 1} f(x) = -1$