Calculus I - MAC 2311 - Section 003

Homework 2

Instructions: Solve the following exercises in a **separate sheet of paper**. Be tidy and organized! You can work on the exercises with your friends (or enemies!) but the final editing has to be yours. The homework has to be returned **by Wednesday October 10, 12:30 pm**. The total number for this homework is 110 (there are 10 extra points). The grade you will receive for this homework will count as a part of *Quizzes and handwritten homework* component of the total grade (15%).

Ex 1. (30 points) Differentiate with respect to the indicated variable. If k appears in the function, treat it as a constant. Before starting computing your derivative, think if it is possible to simplify the function. Show all your work.

a) $\dfrac{d}{dx}\left[\dfrac{x^{11}}{11}+\sqrt[5]{x}+\dfrac{1}{3x^2}+2\right]$

b) $\dfrac{d}{dt}\left[\sqrt[3]{\dfrac{1}{t^5}}\right]$

c) $\dfrac{d}{du}\left[5u\tan(u)\right]$

d) $\dfrac{d}{dx}\left[\dfrac{e^x+1}{\sin(3x)}\right]$

e) $\dfrac{d}{dx}\left[\dfrac{e^\pi+1}{\cos(3\pi)}\right]$

f) $\dfrac{d}{dt}\left[\dfrac{t\ln(t)+t}{t}\right]$

g) $\dfrac{d}{dx}\left[e^{x^2+1}+\ln(\cos(x))\right]$

h) $\dfrac{d}{d\theta}\left[\cos\left(\pi\sqrt{\theta}\right)\right]$

i) $\dfrac{d}{d\alpha}\left[\sqrt{\tan(k\alpha^2)}\right]$

j) $\dfrac{d}{dx}\left[e^{\ln(\sin(x))}\right]$

k) $\dfrac{d}{dt}\left[\ln\left((\sin(t))^{3k}\right)\right]$

l) $\dfrac{d}{dx}\left[x^{\cos(x)}\right]$

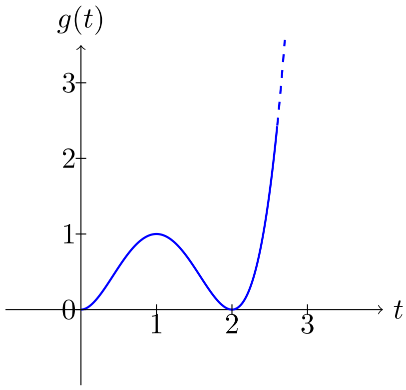
Ex 2. (20 points) At a time $t=0$ a calculus student leaves his home and starts walking toward the university where he has to take his calculus test. At some point he realizes that he has forgotten his calculator at home...

Assume that the student walks/runs according to the position function:

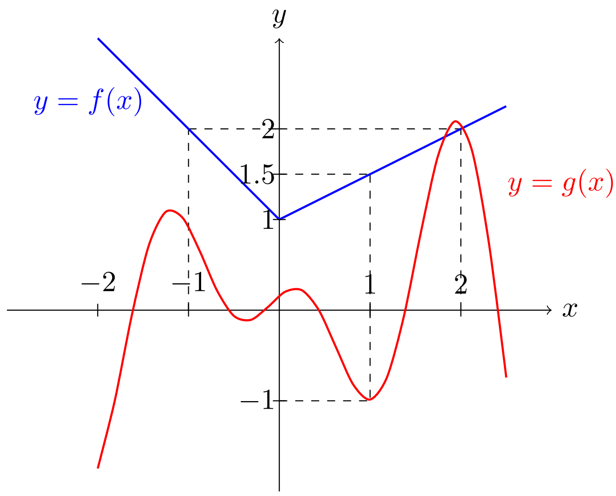
$$g(t)=t^4-4t^3+4t^2,$$

where t is in minutes and $g(t)$ in yards.

a) Find the velocity of the student as a function of t.
b) At what time(s) does he stop?
c) Find the acceleration of the student as a function of t.
d) Find his acceleration at $t=3$ min.
e) Here to the right is the graph of the position function $g(t)$. Is your answer for (b) consistent with this graph? Why?



Ex 3. (20 points)



Let f and g be the functions whose graphs are shown above and let

$$h(x) = f(x) + g(x), \qquad u(x) = f(x)g(x), \qquad v(x) = \frac{f(x)}{g(x)}, \qquad w(x) = f(g(x)).$$

Compute $h'(1)$, $u'(1)$, $v'(1)$ and $w'(1)$, without finding explicit formulæ for $f(x)$ and $g(x)$.

Ex 4. (20 points) The **ideal gas law** relates the temperature, pressure, and volume of an ideal gas. Given n moles of gas, the pressure P (in kPa), volume V (in liters), and temperature T (in kelvin) are related by the equation

$$PV = nRT,$$

where R is the molar gas constant ($R \cong 8.314 \frac{\text{kPa}\cdot\text{ liters}}{\text{kelvin}}$). Assume that the pressure, the volume and the temperature of the gas depend all on time.

a) Suppose that one mole of ideal gas is held in a closed container with a volume of 25 liters. If the temperature of the gas is increasing at a rate of 3.5 kelvin/min, how quickly will the pressure increase?

b) Suppose instead that the temperature of the gas is held fixed at 300 kelvin, while the volume decreases at a rate of 2.0 liters/min. How quickly is the pressure of the gas increasing at the instant that the volume is 20 liters?

Ex 5. (20 points) Which statements are True/False? Justify your answers.

a) If $f(x)$ is a polynomial of degree n then $f^{(n+1)}(x) = 0$.

b) Let $F(t)$ be a physical quantity depending on time. If $\dfrac{dF}{dt}$ is constant for each time t, then F is constant.

c) Let $h(x) = g(f(x))$. If $f'(0) = 1$ and $g'(0) = 0$, then $h'(0) = 0$.

d) We have $\ln(3e^2) - \ln(3\sqrt{e}) = \frac{3}{2}$.