Laws of Exponents and Logarithms

Annamaria Iezzi

Laws of Exponents

Let a, b be positive numbers (> 0) and x, y be real numbers. Then:

- $a^x \cdot a^y = a^{x+y}$
- $a^{-x} = \dfrac{1}{a^x}$
- $\dfrac{a^x}{a^y} = a^{x-y}$
- $(a^x)^y = a^{x \cdot y}$
- $(a \cdot b)^x = a^x \cdot b^x$
- $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$
- If n is a natural number we have:
$$a^n = \underbrace{a \cdot a \cdots a}_{\text{n times}} \qquad \text{and} \qquad a^{\frac{1}{n}} = \sqrt[n]{a}$$
  In particular:
$$a^0 = 1 \qquad \text{and} \qquad a^1 = a$$

Laws of Logarithms

Let a, b be positive numbers (different from 1) and x, y be positive numbers. Then:

- $\log_a(x) = y \Leftrightarrow a^y = x$
- $\log_a(x) + \log_a(y) = \log_a(x \cdot y)$
- $\log_a(x) - \log_a(y) = \log_a\left(\dfrac{x}{y}\right)$
- $\log_a(x^y) = y \log_a(x)$
- $\log_b(x) = \dfrac{\log_a(x)}{\log_a(b)}$ (Change of base formula)
- $\log_a(a) = 1$
- $\log_a(1) = 0$
- $\log_a(a^x) = x$ and $a^{\log_a(x)} = x$ (Cancellation equations).

<u>Notation</u>. $\log(x) := \log_{10}(x)$ and $\ln(x) := \log_e(x)$.