Calculus I - MAC 2311 - Section 007

Quiz 3

09/28/2017

Compute the following derivatives:

1) [2 points] $f(x) = x^5 + 2x^3 - 3x^2 + 1$

Solution:

$$\begin{aligned} f'(x) &= (x^5 + 2x^3 - 3x^2 + 1)' = \\ &= (x^5)' + (2x^3)' - (3x^2)' + (1)' = \\ &= (x^5)' + 2(x^3)' - 3(x^2)' + (1)' = \\ &= 5x^4 + 2 \cdot 3x^2 - 3 \cdot 2x + 0 = \\ &= 5x^4 + 6x^2 - 6x. \end{aligned}$$

2) [2 points] $f(x) = x^{2017} - 2018 \cos x$

Solution:

$$\begin{aligned} f'(x) &= (x^{2017} - 2018 \cos x)' = \\ &= (x^{2017})' - (2018 \cos x)' = \\ &= 2017x^{2016} - 2018(\cos x)' = \\ &= 2017x^{2016} - 2018(-\sin x) = \\ &= 2017x^{2016} + 2018 \sin x. \end{aligned}$$

3) [2.5 points] $f(x) = x^2 \tan x$

Solution:

$$\begin{aligned} f'(x) &= (x^2 \tan x)' = \\ &= (x^2)' \tan x + x^2 (\tan x)' = \\ &= 2x \tan x + x^2 \cdot (1 + \tan^2 x) = \\ &= 2x \tan x + x^2 + x^2 \tan^2 x = \\ &= x(x \tan^2 x + 2 \tan x + x). \end{aligned}$$

Also the solution $f'(x) = 2x \tan x + \dfrac{x^2}{\cos^2 x} = 2x \tan x + x^2 \sec^2 x$ *is correct if we choose* $(\tan x)' = \dfrac{1}{\cos^2 x} = \sec^2 x$.

4) [2.5 points] $f(x) = 3 \cdot \frac{1+x}{1-x}$

Solution:

$$\begin{aligned} f'(x) &= \left(3 \cdot \frac{1+x}{1-x}\right)' = \\ &= 3 \cdot \left(\frac{1+x}{1-x}\right)' = \\ &= 3 \cdot \frac{(1+x)'(1-x) - (1+x)(1-x)'}{(1-x)^2} = \\ &= 3 \cdot \frac{1 \cdot (1-x) - (1+x) \cdot (-1)}{(1-x)^2} = \\ &= 3 \cdot \frac{1-x+1+x}{(1-x)^2} = \\ &= 3 \cdot \frac{2}{(1-x)^2} = \\ &= \frac{6}{(1-x)^2}. \end{aligned}$$

5) [2.5 points] $f(x) = x \cos x \sin x$

Solution:

$$\begin{aligned} f'(x) &= (x \cos x \sin x)' = \\ &= ((x \cos x)(\sin x))' = \\ &= (x \cos x)' \sin x + x \cos x (\sin x)' = \\ &= ((x)' \cos x + x(\cos x)') \sin x + x \cos x \cos x = \\ &= (\cos x + x(-\sin x)) \sin x + x \cos^2 x = \\ &= \cos x \sin x - x \sin^2 x + x \cos^2 x = \\ &= \cos x \sin x + x(\cos^2 x - \sin^2 x) = \\ &= \frac{1}{2} \sin(2x) + x \cos(2x). \end{aligned}$$

As f is the product of three functions (x, $\cos x$ and $\sin x$), we can apply before the product rule to the functions $x \cos x$ ans $\sin x$ and then again to the functions x and $\cos x$. Of course we would have obtained the same result if we had applied the product rule to the functions x ans $\cos x \sin x$ and then to the functions $\cos x$ and $\sin x$.

In the last step we used the trigonometric formulas:

$$\sin(2x) = 2 \sin x \cos x$$
$$\cos(2x) = \cos^2 x - \sin^2 x.$$