Calculus I - MAC 2311 - Section 007

Quiz 2 - Solution

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1) [3 points] Give the definition of a function $f : \mathbb{R} \mapsto \mathbb{R}$ continuous at a point a in $\mathbb{R}$.

Solution:

A function $f : \mathbb{R} \to \mathbb{R}$ *is said to be* continuous *at* $a \in \mathbb{R}$ *if*

$$\lim_{x \to a} f(x) = f(a).$$

2) [3 points] Let $f : \mathbb{R} \mapsto \mathbb{R}$ be a function and a a point such that

$$\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = L < \infty \text{ and } f(a) \neq L.$$

How do we call this kind of discontinuity?

Solution:

It is a removable *discontinuity.*

Find a function g that agrees with f for all $x \neq a$ and is continuous at a.

Solution:

$$g = \begin{cases} f(x), & \text{for } x \neq a \\ L, & \text{for } x = a \end{cases}$$

2) [5 points] For which values of x is the following function continuous? For each discontinuity establish if it is a removable, an infinite or a jump discontinuity.

$$f(x) = \frac{x-2}{x^2 - 3x + 2}.$$

Solution:

First of all we can write

$$f(x) = \frac{x-2}{(x-1)(x-2)}.$$

Hence the domain of f *is the set* $D = \mathbb{R} \setminus \{1, 2\}$.

Since $f(x)$ *is a rational function, it is continuous at each point of its domain, so that we will just analyze the behavior of* f *at a neighborhood of* $x = 1$ *and* $x = 2$. *We have:*

- $\displaystyle\lim_{x \to 1^-} \frac{x-2}{(x-1)(x-2)} = \lim_{x \to 1^-} \frac{1}{x-1} = \text{“}\frac{1}{0^-}\text{”} = -\infty$ and $\displaystyle\lim_{x \to 1^+} \frac{x-2}{(x-1)(x-2)} = \lim_{x \to 1^+} \frac{1}{x-1} = \text{“}\frac{1}{0^+}\text{”} = \infty$. *Hence* $x = 1$ *is an infinite discontinuity.*
- $\displaystyle\lim_{x \to 2^-} \frac{x-2}{(x-1)(x-2)} = \lim_{x \to 2^-} \frac{1}{x-1} = 1$ and $\displaystyle\lim_{x \to 2^+} \frac{x-2}{(x-1)(x-2)} = \lim_{x \to 2^+} \frac{1}{x-1} = 1$. *Moreover* f *is not defined at* 2, *hence* $x = 2$ *is a removable discontinuity.*