Bridge - MGF 3301 - Section 001

TEST 2

03/11/2020

Print your name and sign below, and read the instructions. Do not open the test until you are told to do so.

Name:	
U number:	
Signature:	

Instructions

This test contains 7 exercises. The total number of points is 120, but your grade will be the minimum between your score and 110 (you can get up to 10 bonus points). Calculators are not allowed (and actually not needed).

Put all your answers in the spaces provided on these sheets. The last sheet of the test is blank and may be used for scratch work. More scratch paper is available on request.

In each proof you may use a result proved in class, in the homework or in previous exercises of this test. In this case just state clearly which result you are using, but there is no need of proving it again.

Neatness and clarity are important. You will lose credit if we cannot decipher your answer.

Do not write in this table

1		5	
2		6	
3		7	
4		**TOTAL**	

Exercise 1 (27 points)

Consider the following set:

$$A = \{\{\varnothing\}, 3, \{3, 4\}\},$$

(a) [12 points] True or false?

a1) $3 \in A \cap 3\mathbb{Z}$

☐ TRUE

☐ FALSE

a2) $\{\{\varnothing\}, 3\} \in A$

☐ TRUE

☐ FALSE

a3) $\{3, 4\} \subseteq A$

☐ TRUE

☐ FALSE

a4) $\varnothing \in A$

☐ TRUE

☐ FALSE

a5) $\{\varnothing\} \in A$

☐ TRUE

☐ FALSE

a6) $\varnothing \subseteq \mathcal{P}(A)$

☐ TRUE

☐ FALSE

(b) [3 points] Explain briefly and fully your answer for one (and only one) among a1, a2, a3, a4, a5, a6.

The explanation for

☐ a1 ☐ a2 ☐ a3 ☐ a4 ☐ a5 ☐ a6

is...

(c) [13 points] List all the elements of the following sets:

c1) $A \cap \mathbb{N} =$

c2) $A \cup \{\varnothing, 1, 2, 3, 4\} =$

c3) $A \setminus \mathbb{Q} =$

c4) $\mathcal{P}(A) =$

c5) $A \cap \mathcal{P}(A) =$

Exercise 2 (16 points)

Describe the following sets with a *set-builder notation*, i.e. as truth set of an open sentence. Remember that we use the convention that $\mathbb{N} = \{1, 2, 3, \ldots\}$.

a) $A = \{\ldots, -2, 0, 2, 4, 6, 8 \ldots\ldots\}$

b) $B = \{0, 10, 20, 30, 40, \ldots, 1000\}$

c) $C = \left\{1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots\right\}$

d) $D = \left\{\cdots - \frac{5}{4}, -\frac{4}{3}, -\frac{3}{2}, -\frac{2}{1}\right\}$

Exercise 3 (18 points)

(a) [10 points] Prove **by contrapositive** the following claim (please, write down the contrapositive of the statement first):

Claim 1: *Let $n \in \mathbb{Z}$.* If *n^3 is even* then *n is even.*

(b) [8 points] Prove the following claim:

Claim 2: *For all* $n \in \mathbb{Z}$, n *is even* if and only if n^3 *is even.*

You may use Claim 1 for some part of the proof. In this case, no need of proving Claim 1 again.

Exercise 4 (26 points)

(a) [7 points] Let A, B, C be sets. Prove that

$$C \subseteq A \cap B \Rightarrow C \subseteq A \text{ and } C \subseteq B.$$

(b) [7 points] Prove that $\mathcal{P}(A \cap B) \subseteq \mathcal{P}(A) \cap \mathcal{P}(B)$.

(c) [6 points] Diego says:

"*If* $C \subseteq A \cup B$ *then* $C \subseteq A$ *or* $C \subseteq B$."

Do you agree with Diego? If yes prove his statement, otherwise show a counterexample.

(d) [6 points] Build an example of two sets A and B such that $\mathcal{P}(A \cup B) \nsubseteq \mathcal{P}(A) \cup \mathcal{P}(B)$.

Exercise 5 (13 points)

Prove **by induction** that the sum of the first n natural even integers is equal to $n(n+1)$, i.e. prove that

$$\forall\, n \in \mathbb{N},\ 2 + 4 + \cdots + 2n = n(n+1).$$

Exercise 6 (10 points)

Prove **by contradiction** the following claim:

Claim 3: $\sqrt[3]{2}$ *is irrational.*

If at some point of your proof you make use of any result proved in class, in a homework or in a previous exercise of this test, please state it (but there is no need of proving it again).

Exercise 7 (10 points)

Let $a, b \in \mathbb{Z}$. Prove that if $4 \mid (a^2 + b^2)$ then a is even or b is even.