Bridge - MGF 3301 - Section 001

TEST 1

02/12/2020

Print your name and sign below, and read the instructions. Do not open the test until you are told to do so.

Name:	
U number:	
Signature:	

Instructions

This test contains 6 exercises. The total number of points is 110 (there are 10 bonus points). Calculators are not allowed (and actually not needed).

Put all your answers in the spaces provided on these sheets. The last sheet of the test is blank and may be used for scratch work. More scratch paper is available on request.

Neatness and clarity are important. You will lose credit if we cannot decipher your answer.

Do not write in this table

1		4	
2		5	
3		6	

Exercise 1 (10 points)

Consider the following propositions:

- ▶ $P :=$ "An even number can be written in the form $2k + 1$, for some integer k."
- ▶ $Q :=$ "A square has 5 sides."
- ▶ $R :=$ "There exists a rational number x such that $0 < x < \frac{1}{2}$."

a) [2 points] The **truth value** of P is

☐ TRUE

☐ FALSE

b) [2 points] The **truth value** of Q is

☐ TRUE

☐ FALSE

c) [2 points] The **truth value** of R is

☐ TRUE

☐ FALSE

d) [4 points] For the above propositions P, Q and R, determine the **truth value** of the propositional form

$$(P \wedge (\sim Q)) \Rightarrow (R \vee P).$$

Justify your answer.

Exercise 2 (15 points)

Write a **non-trivial denial** (i.e. not of the form *It is not the case that...*, *There exists no...*, *Not all...*, *etc.*) of the following propositions:

a) *x is even and y is divisible by 3.*

b) *The function f has a local maximum at $x = -1$ or at $x = 1$.*

c) $\forall x$ *in* $\mathbb{R}$, x^2 *is a rational number.*

d) $\exists n$ *in* $\mathbb{Z}$ *such that* $n + 1$ *is a prime number.*

e) $\exists x$ *in* $\mathbb{R}$ *such that* $\forall y$ *in* $\mathbb{R}$ $xy = 0$.

Exercise 3 (33 points)

For x a real number, let $P(x)$ and $Q(x)$ be the following open sentences:

$$P(x) := \text{“}x^2 - 3x + 2 = 0,\text{”} \qquad Q(x) := \text{“}x \geq 0.\text{”}$$

a) [3 points] Determine the **truth set** of $P(x)$. Justify your answer.

b) [2 points] Determine the **truth set** of $Q(x)$. (Write it as an interval.)

c) [3 points] Determine the **truth set** of $P(x) \vee Q(x)$. Justify your answer.

d) [4 points] Determine the **truth value** of the proposition “$\exists!\, x$ in $\mathbb{R}$ such that $P(x) \wedge Q(x)$”. Justify your answer.

e) Consider now the conditional sentence:

$$R(x) := \text{“}P(x) \Rightarrow Q(x)\text{”} = \text{“}\textit{If } x^2 - 3x + 2 = 0 \textit{ then } x \geq 0.\text{”}$$

- [3 points] What is the **truth value** of $R(0)$? Justify your answer.

- [3 points] What is the **truth value** of $R(1)$? Justify your answer.

- [5 points] What is the **truth value** of "$\forall x,\ R(x)$"? Justify your answer.

- [5 points] Write the **converse** of $R(x)$. Is it true for all x in $\mathbb{R}$? Justify your answer.

- [5 points] Write the **contrapositive** of $R(x)$. Is it true for all x in $\mathbb{R}$? Justify your answer.

Exercise 4 (29 points)

a) [9 points] Write the following **definitions**:

- An integer n is said to be *even* if and only if...

- An integer n is said to be *odd* if and only if...

- Let n, m be integers. The integer n is said to be *divisible by* m if and only if...

b) [10 points] Prove that the following two propositional forms are **equivalent**:

$$P \Rightarrow (Q \wedge R) \quad \text{and} \quad (P \Rightarrow Q) \wedge (P \Rightarrow R).$$

c) [10 points] **Prove** the following claim:

Let n be an integer. If *n is even* then *$n + 1$ is odd and n^2 is divisible by 4.*

Exercise 5 (13 points)

Consider the following definition:

Definition

An integer n is said to be a **perfect square** if there exists an integer k such that $n = k^2$.

a) [3 points] Give **three** different **examples** of integers that are perfect squares.

b) [10 points] Use the above definition to **prove** the following claim:

Let n and m be two integers. If *n and m are perfect squares,* then *also their product is a perfect square.*

Exercise 6 (10 points) – **Who is telling the truth?**

During the interrogatory of a trial, Anna, Diego and Vanessa made the following statements:

Anna

"Exactly two people (among Diego, Vanessa and me) are lying."

Diego

"Vanessa or I are telling the truth."

Vanessa

"Diego is telling the truth."

In front of these declarations, the trial judge feels the need of consulting a specialist. They call then Andrea, a student in Bridge to Abstract Mathematics, in order to complete the investigation. After thinking for a while, Andrea says with regret to the judge:

Andrea: *"Unfortunately, I do not have enough information in order to uniquely identify all the people who are telling the truth."*

Do you agree with your classmate Andrea? Explain fully and concisely why or why not.