

THE TERMINOLOGY OF A MATH PAPER

- **DEFINITION**

A precise and unambiguous description of the meaning of a mathematical term. It characterizes the meaning of a word by giving all the properties and only those properties that must be true.

- **THEOREM**

A mathematical statement that is proved using rigorous mathematical reasoning. In a mathematical paper, the term theorem is often reserved for the most important results that says something definitive on the subject.

- **PROPOSITION**

A proved and often interesting result, but generally less important than a theorem.

- **LEMMA**

A minor result whose sole purpose is to help in proving a theorem, because it is usually not particularly interesting in its own right. It is a stepping stone on the path to proving a theorem. Very occasionally lemmas can take on a life of their own (Zorn's lemma, Urysohn's lemma, Burnside's lemma, Sperner's lemma).

- **COROLLARY**

A result in which the (usually short) proof relies heavily on a given theorem (we often say that "this is a corollary of Theorem A").

- **CLAIM**

An assertion that is then proved. It is often used like an informal lemma.

- **CONJECTURE**

A statement that is unproved, but is believed to be true (Collatz conjecture, Goldbach conjecture, twin prime conjecture).

- **AXIOM/POSTULATE**

A statement that is assumed to be true without proof. These are the basic building blocks from which all theorems are proved (Euclid's five postulates, Zermelo-Frankel axioms, Peano axioms).

- **IDENTITY**

A mathematical expression giving the equality of two (often variable) quantities (trigonometric identities, Euler's identity).

- **PARADOX**

A statement that can be shown, using a given set of axioms and definitions, to be both true and false. Paradoxes are often used to show the inconsistencies in a flawed theory (Russell's paradox). The term paradox is often used informally to describe a surprising or counterintuitive result that follows from a given set of rules (Banach-Tarski paradox, Alabama paradox, Gabriel's horn).

"Note that from a logical point of view, there is no difference between a theorem, proposition, lemma and corollary. They are all claims waiting to be proved. However, we use these terms to suggest different levels of importance and difficulty." (Terence Tao – Analysis I, p. 25, n. 4)

This definition sheet is largely inspired by some notes of Prof. Dave Richeson that you can find at the following link:

<https://divisbyzero.com/2008/09/22/what-is-the-difference-between-a-theorem-a-lemma-and-a-corollary/>.