

Name and surname:

## Bridge - MGF 3301 - Section 001

### Quiz 6

04/08/2020

**Instructions:** The total number of points for this quiz is 12 (there are 2 bonus points), however, your final score will be the minimum between the total number of your points and 11. Calculators are not allowed (and actually not needed).

**When you have completed your work, please submit it by 10am on Gradescope.com, under the assignment *Quiz 6*. Remember that you have to submit **one unique pdf**.**

#### EXERCISE 1

(12 points)

Consider the following relation on  $\mathbb{Z}$ :

$$R = \{(a, b) \in \mathbb{Z}^2 : 3 \mid (a - b)\}.$$

(a) (2 points) Which ordered pairs among the following belong to  $R$ ? Select all that apply.

(23,17)

(17,23)

(18,17)

(17,17)

(b) (2 points) Prove that  $R$  is reflexive on  $\mathbb{Z}$ .

(c) (2 points) Prove that  $R$  is symmetric.

(d) (2 points) Prove that  $R$  is transitive.

Recall that for  $a \in \mathbb{Z}$ , we denote  $\bar{a} := \{b \in \mathbb{Z} : (a, b) \in R\}$ .

(e) (2 points) Prove that, for the relation  $R$  defined previously, we have  $\bar{0} = 3\mathbb{Z}$ .

(f) (2 points) Prove that if  $x \in \bar{1}$  then  $x = 3k + 1$ , for some integer  $k$ .