Bridge - MGF 3301 - Section 001

Quiz 5 - Solution

03/04/2020

Instructions: The total number of points for this quiz is 11 (there is 1 bonus point). Calculators are not allowed (and actually not needed).

EXERCISE 1
(6 points)

Describe the following sets with a *set-builder notation*, i.e. as truth set of an open sentence.

(a) $A = \{2, 3, 5, 7, 11, 13, \ldots\}$

Solution

$A = \{n \in \mathbb{N} : n \text{ is prime}\}$.

(b) $B = \{1, 3, 5, 7, \ldots, 49\}$

Solution

$B = \{n \in \mathbb{N} : n = 2k + 1,\ k \in \mathbb{Z} \text{ and } 0 \leq k \leq 24\}$.

(c) $C = \left\{\frac{1}{5}, \frac{1}{10} \frac{1}{15}, \frac{1}{20}, \ldots\right\}$

Solution

$C = \left\{x \in \mathbb{Q} : x = \frac{1}{5k},\ k \in \mathbb{N}\right\}$.

(d) $D = \left\{\frac{1}{5}, \frac{2}{10} \frac{3}{15}, \frac{4}{20}, \ldots\right\}$

Solution

$D = \left\{x \in \mathbb{Q} : x = \frac{k}{5k},\ k \in \mathbb{N}\right\}$.

Exercise 2
(5 points)

Let $a \in \mathbb{Z}$. Recall the following notation:

$$a\mathbb{Z} := \{n \in \mathbb{Z} \,|\, n = ak, k \in \mathbb{Z}\}.$$

(a) Prove that $6\mathbb{Z} \subseteq 3\mathbb{Z}$.

Solution

Proving that $6\mathbb{Z} \subseteq 3\mathbb{Z}$ is equivalent to prove that if $n \in 6\mathbb{Z}$, then $n \in 3\mathbb{Z}$. Let $n \in 6\mathbb{Z}$. Then there exists $k \in \mathbb{Z}$ such that $n = 6k \Rightarrow n = 3 \cdot (2k)$. So n is also a multiple of 3, which implies that $n \in 3\mathbb{Z}$.

(b) Prove that $6\mathbb{Z} \neq 3\mathbb{Z}$.

Solution

It is enough to show that $3\mathbb{Z} \not\subseteq 6\mathbb{Z}$. For that, note that $3 \in 3\mathbb{Z}$ (since $3 = 3 \cdot 1$ is a multiple of 3), but $3 \notin 6\mathbb{Z}$. Indeed if, to the contrary, $3 \in 6\mathbb{Z}$, then $\exists k \in \mathbb{Z}$ such that $3 = 6k \Rightarrow \frac{1}{2} = k$, which is a contradiction with the fact that k is an integer.