# Bridge - MGF 3301 - Section 001

## Quiz 3 - Solution

02/05/2020

**Instructions:** The total number of points for this quiz is 10. You will get an extra point if you solve correctly the last exercise. Calculators are not allowed (and actually not needed).

### Exercise 1

(4 points)

Consider the following open sentence:

$$P(x) = \text{“}x^2 - 5x + 4 = 0 \text{ } \textit{and} \text{ } 0 < x < 3.\text{”}$$

(a) If the Universe is $\mathbb{R}$, what is the truth set of $P(x)$?

**Solution**

We can write $P(x) = Q(x) \wedge R(x)$, where $Q(x) = \text{“}x^2 - 5x + 4 = 0\text{”}$ and $R(x) = \text{“}0 < x < 3.\text{”}$ So the truth set of $P(x)$ will be the intersection of the truth set of $Q(x)$ and the truth set of $R(x)$.

Now, the truth set of $Q(x)$ is $\{1, 4\}$, since $x = 1$ and $x = 4$ are all and the only solutions of the equation $x^2 - 5x + 4 = 0$. The truth set of $R(x)$ is given by the real interval $(0, 3)$.

In conclusion we get that the truth set of $P(x)$ is $\{1, 4\} \cap (0, 3) = \{1\}$.

(b) If the Universe is $\mathbb{R}$, what is the truth set of $\sim P(x)$?

**Solution**

Since $\sim P(x)$ is true exactly when $P(x)$ is false, the truth set of $\sim P(x)$ is given by the complementary of the truth set of $P(x)$, i.e. all the real numbers but $x = 1$, which we can write:

$$\mathbb{R} \setminus \{1\}.$$

### Exercise 2

(6 points)

Consider the following (open) conditional sentence in the variables $a$ and $x$, where $a$ and $x$ are real numbers:

$$P(a, x) = \text{“}ax = 0 \text{ } \textit{and} \text{ } x \neq 0 \Rightarrow a = 0.\text{”}$$

(a) What is the truth value of $P(0,1)$? What about $P(1,0)$? Justify your answer.

**Solution**

$P(0,1)$ is true, since for $a = 0$ and $x = 1$ we have that the antecedent is false.

$P(1,0)$ is also true, since for $a = 1$ and $x = 0$ we have that the antecedent and the consequent are both true.

(b) Write the converse of $P(a,x)$.

**Solution**

The converse of $P(a,x)$ is:

$$\text{“}a = 0 \Rightarrow ax = 0 \text{ and } x \neq 0.\text{”}$$

(c) Write the contrapositive of $P(a,x)$.

**Solution**

The converse of $P(a,x)$ is:

$$\text{“}a \neq 0 \Rightarrow ax \neq 0 \text{ or } x = 0.\text{”}$$

## Exercise 3
(Bonus - 1 point)

Is the following statement true?

$$x \text{ is a butterfly} \Leftrightarrow x \text{ is an insect.}$$

Justify your answer.

**Solution**

The statement is false, because the conditional sentence

$$x \text{ is an insect} \Rightarrow x \text{ is a butterfly.}$$

is not true for every $x$. Indeed a cockroach is an example of insect which is not a butterfly (so for $x =$*cockroach* you would have that the antecedent is true while the consequent is false, making false the overall conditional sentence).