Bridge - MGF 3301 - Section 001

Homework 9

Instructions

Please read and follow this instructions carefully, otherwise we will not be able to grade your work.

This homework contains 3 exercises. Keep in mind that at the end **you will have to submit a pdf** (no .png, .jpg, etc.). For that you will have three options:

a) If you have a **tablet with a stylus**, write your answers to the exercises directly on this pdf, in the provided blank spaces. When you have completed your work, save it as a pdf.

b) If you do not have a tablet with a stylus, but you do have access to a **printer**, print this pdf and write your answers to the exercises in the provided blank spaces. When you have completed your work, scan it with a printer or with a smartphone (in the latter case, you will need a **scanner app**, I personally use *Tiny scanner*)

c) If you have neither a tablet, nor a printer, solve as usual these exercises on a separate sheet of paper. When you have completed your work, scan it with your smartphone (you will need a **scanner app**, I personally use *Tiny scanner*).

Once you have your pdf, please submit it on `Gradescope.com` under the assignment *Homework 9*.

If you have any doubt about the submission process, please ask me (via the chat of MS Teams or via email) before proceeding.

As usual you can work on the exercises with your friends (or enemies!) but the final editing has to be yours. This homework has to be submitted **by Wednesday April 8 at 9:30 am**. The total number for this homework is 110 (there are 10 extra points). The grade you will receive for this homework will count as a part of *Homework* component of the total grade (15%).

Ex 1. [34 points total] Consider the following relation on $\mathbb{Z}$:

$$R = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x + y = 6\}$$

(1a) (10 points) Write 5 different elements of R.

(1b) (8 points) Is R reflexive on $\mathbb{Z}$?

☐ YES

☐ NO

Justify your answer:

(1c) (8 points) Is R symmetric?

☐ YES

☐ NO

Justify your answer:

(1d) (8 points) Is R transitive?

☐ YES

☐ NO

Justify your answer:

Ex 2. [42 points total] Let A be the following set of words:

$A = \{\text{bridge, link, overpass, platform, arch, branch, connection, viaduct, bond, catwalk}\}$

Consider the relation on A, such that, for $x, y \in \mathbb{A}$,

$$x \sim y \Leftrightarrow x \text{ and } y \text{ have the same number of letters.}$$

(2a) (15 points) Prove that $\sim$ is an equivalence relation, i.e. prove that it is reflexive, symmetric and transitive.

(2b) (10 points) For each equivalence class of R list all its elements, i.e. write down the subsets $\overline{x} = \{y \in A : x \sim y\}$, where $x \in A$.

(2c) (10 points) Write all the elements of the set A/R (A modulo R).

(2d) (7 points) Is the following statement true or false?

$$\textit{Let } x, y \in A. \text{ If } y \notin \overline{x}, \text{ then } \overline{x} \cap \overline{y} = \varnothing.$$

☐ TRUE

☐ FALSE

Justify your answer:

Ex 3. **[34 points total]** Note that in this exercise each part is independent of the others.

(3a) (14 points) Find the domain and the range of the following relation:

$$R = \{(x, y) \in \mathbb{R}^2 : x^2 > 1 \text{ and } y > 2\}.$$

Justify your answer. (*Suggestion: you may find this question easier once you have interpreted the relation R geometrically in the real plane.)*

(3b) (10 points) Let R be a relation from A to B and S a relation from B to C. Using the definition of the *domain* of a relation and the definition of the *composite* of two relations, prove that:

$$\textit{if } a \in \mathrm{Dom}(S \circ R) \textit{ then } a \in \mathrm{Dom}(R).$$

(3c) (10 points) Let R be a relation on A. Using the definition of the *symmetric property* and the definition of the *inverse relation*, prove that if R is symmetric, then $R^{-1} = R$.