

Bridge - MGF 3301 - Section 001

Homework 8

INSTRUCTIONS

Please read and follow this instructions carefully, otherwise we will not be able to grade your work.

This homework contains 3 exercises. Keep in mind that at the end **you will have to submit a pdf** (no .png, .jpg, etc.). For that you will have three options:

- a) If you have a **tablet with a stylus**, write your answers to the exercises directly on this pdf, in the provided blank spaces. When you have completed your work, save it as a pdf.
- b) If you do not have a tablet with a stylus, but you do have access to a **printer**, print this pdf and write your answers to the exercises in the provided blank spaces. When you have completed your work, scan it with a printer or with a smartphone (in the latter case, you will need a **scanner app**, I personally use *Tiny scanner*)
- c) If you have neither a tablet, nor a printer, solve as usual these exercises on a separate sheet of paper. When you have completed your work, scan it with your smartphone (you will need a **scanner app**, I personally use *Tiny scanner*).

Note that the **instructions for submission** (through Gradescope) will be given soon, in a separate announcement.

If you have any doubt about the submission process, please ask me (via the chat of MS Teams or via email) before proceeding.

As usual you can work on the exercises with your friends (or enemies!) but the final editing has to be yours. This homework has to be submitted **by Wednesday April 1 at 9:30 am**. The total number for this homework is 110 (there are 10 extra points). The grade you will receive for this homework will count as a part of *Homework* component of the total grade (15%).

Ex 1. [30 points total] Consider the following sets:

$$A = \{\emptyset, 1, \{1\}\}, \quad B = \{1, 2, 3\}, \quad C = \{2, 3, 4\}.$$

(1a) (20 points) List all the elements of the following sets:

(a) $A \times B =$

(b) $A \times C =$

(c) $(A \times B) \cap (A \times C) =$

(d) $A \times (B \cap C) =$

(1b) (10 points) You have probably noticed that in the example above sets (c) and (d) are equal. Prove this fact in general, i.e. prove that given three sets A , B and C we have

$$A \times (B \cap C) = (A \times B) \cap (A \times C).$$

(Hint: prove both inclusions, i.e. prove that $A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$ and $(A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$).

Ex 2. [35 points total] An important theorem in arithmetic says the following:

Theorem/Definition: Division Algorithm

For all integers a and b , with $a \neq 0$, there exist unique integers q and r such that

$$a = bq + r \text{ and } 0 \leq r < |b|.$$

The integer a is called **dividend**, b the **divisor**, q the **quotient** and r is the **remainder**.

Example: If $a = 23$ and $b = 7$, then $q = 3$ and $r = 2$, since

$$23 = 7 \cdot 3 + 2.$$

Note that $r = 2$ satisfies $0 \leq r < 7$.

(2a) (5 points) Determine the **quotient** q and the **remainder** r for $a = 2020$ and $b = 7$.

(2b) Consider now the following sets:

$$A = \{0, 1, 2, \dots, 8\}, \quad B = \{0, 1, 2\},$$

and the relation defined as follows. Given $x \in A$ and $y \in B$,

$$x \sim y \Leftrightarrow \text{the remainder of the division of } x \text{ by } 3 \text{ is equal to } y.$$

(2b.1) (15 points) Write the subset $R \subseteq A \times B$ that corresponds to the above relation from A to B .

(2b.2) (15 points) Draw the *arrow diagram* that represents the relation R from A to B .

Ex 3. [45 points total] Let A be the set

$$A = \left\{ \frac{22}{7}, \pi, 3.14, \sqrt{10} \right\}.$$

(3a) (10 points) List all the elements of $A^2 := A \times A$.

(3b) Consider the following relation on A . Given $x, y \in A$,

$$x \sim y \Leftrightarrow x < y.$$

(3b.1) (15 points) Write the subset $R \subseteq A^2$ that corresponds to the above relation.

(3b.2) (15 points) Draw the *directed graph* that represents the relation R on A .

(3b.3) (5 points) Do there exist $x, y \in A$ such that $(x, y) \in R$ and $(y, x) \in R$? Why or why not?