Bridge - MGF 3301 - Section 001

Homework 7

Instructions: Solve **Exercise 2** on this sheet and all the others on a **separate sheet of paper**. Be tidy and organized! You can work on the exercises with your friends (or enemies!) but the final editing has to be yours. This homework has to be returned **by Wednesday March 11 at 9:30 am**. The total number for this homework is 110 (there are 10 extra points). The grade you will receive for this homework will count as a part of *Homework* component of the total grade (15%).

Ex 1. [26 points total] Consider the following sets:

$$A = \{\varnothing, 0, 1, \{1, 2\}\},$$
$$B = \{\{\varnothing\}, 1, 2, 3\},$$
$$C = \mathbb{Z}.$$

List all the elements of the following sets:

(a) $A \cup B$,
(b) $A \cap B$,
(c) $B \setminus A$,
(d) $A \setminus B$,
(e) $(A \cup B) \cap C$,
(f) $\mathcal{P}(B)$, the power set of B,
(g) $(B \setminus C) \cap A$.
(h) $A \cap (C \setminus B)$.

Ex 2. [14 points total] In the following, A and B are sets and x, y are elements. True or false?

$A \subseteq \mathcal{P}(A)$	$A \in \mathcal{P}(A)$	$\{3\} \subseteq \mathcal{P}(\{1,2,3\})$	$3 \in \mathcal{P}(\{1,2,3\})$
☐ TRUE	☐ TRUE	☐ TRUE	☐ TRUE
☐ FALSE	☐ FALSE	☐ FALSE	☐ FALSE

$\{\varnothing\} \in \mathcal{P}(A) \Leftrightarrow \varnothing \in A$	If $B \in \mathcal{P}(A)$ then $B \subseteq A$	If $\{x, y\} \in \mathcal{P}(A)$ then $x, y \in A$
☐ TRUE	☐ TRUE	☐ TRUE
☐ FALSE	☐ FALSE	☐ FALSE

Ex 3. [40 points total] Let A, B, C, D be sets. Recall that proving that "$A \subseteq B$" is equivalent to prove that "if $x \in A$ then $x \in B$".

(a) (10 points) Prove that if $A \subseteq B \cup C$ and $A \cap B = \varnothing$, then $A \subseteq C$.

(b) (10 points) Prove that if $A \subseteq B$, then $\mathcal{P}(A) \subseteq P(B)$.

(c) (10 points) Is the converse of (b) true? If yes prove it, otherwise show a counterexample.

(d) (10 points) Prove by contradiction that if $C \subseteq A \cap B$ and $D \subseteq A \setminus B$ then C and D are disjoint.

Ex 4. [30 points total]

(a) (15 points) Prove by induction that

$$\forall n \in \mathbb{N},\ 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

(b) (15 points) Prove by induction that $2^n > 2n$ for every natural number $n \geq 3$.