Bridge - MGF 3301 - Section 001

Homework 4

Instructions: Solve the following exercises in a **separate sheet of paper**. Be tidy and organized! You can work on the exercises with your friends (or enemies!) but the final editing has to be yours. This homework has to be returned **by Wednesday February 12 at 9:30 am**. The total number for this homework is 110 (there are 10 extra points). The grade you will receive for this homework will count as a part of *Homework* component of the total grade (15%).

Ex 1. [15 points total] Write a non trivial **denial** of the following propositions:

1.a) (5 points) $\forall\, x$ in $\mathbb{R}, x > 0$;

1.b) (5 points) $\exists\, n$ in $\mathbb{N}$ such that n is prime and n is divisible by 6;

1.c) (5 points) $\forall\, x$ in $\mathbb{R}$, $\exists\, a, b \in \mathbb{Z}$ such that $x = \frac{a}{b}$.

Ex 2. [20 points total] Determine the **truth value** of the following propositions (justify your answers):

2.a) (5 points) $\forall\, x$ in $\mathbb{R}$, $x^2 + 1 \geq 0$;

2.b) (5 points) $\forall\, n$ in $\mathbb{N}$, $n^2 + 3n + 2 \neq 0$;

2.c) (5 points) $\exists\, a, b, c$ in $\mathbb{N}$, $a^2 + b^2 = c^2$;

2.d) (5 points) $\forall\, y$ in $\mathbb{R}$, $\exists\, x$ in $\mathbb{R}$ such that $x^2 = y$.

Ex 3. [45 points total]

3.a) (15 points) Let n be an integer. Prove that if n is odd, then n^2 is also odd.

3.b) (15 points) Let x and y be integers. Prove that if x is even and y is divisible by 3, then the product xy is divisible by 6.

3.c) (15 points) Let a and b be real numbers. Prove that if $0 < b < a$, then $a^2 - ab > 0$.

Ex 4. [30 points total]

(4.a) (15 pts) Prove that the following propositional forms are equivalent:

$$(P \vee Q) \Rightarrow R \quad \text{and} \quad (P \Rightarrow R) \wedge (Q \Rightarrow R).$$

Note that this fact tells you that proving that "(P or Q) implies R" is equivalent to prove that "P implies R and Q implies R".

(4.b) Consider now the following statement:

"If n is an integer, then $n^2 - 5n + 2$ is even."

Since an integer can be even or odd, it is easy to see that this statement is equivalent to:

"If n is even or n is odd, then $n^2 - 5n + 2$ is even."

(4.b1) (5 pts) Show that this last statement is of the form $(P \vee Q) \Rightarrow R$, by saying which are the propositions P, Q and R in this case.

(4.b2) (10 pts) Prove the statement, by using the equivalence stated in (4.a), i.e. prove that $(P \Rightarrow R)$ and $(Q \Rightarrow R)$. (This is nothing but an example of **proof by exhaustion**, which consists of an examination of every possible case.)