# Bridge - MGF 3301 - Section 001
# The Last Homework

## Instructions

Please read and follow this instructions carefully, otherwise we will not be able to grade your work.

This homework contains 5 exercises. Keep in mind that at the end **you will have to submit a pdf** (no .png, .jpg, etc.). For that you will have three options:

a) If you have a **tablet with a stylus**, write your answers to the exercises directly on this pdf, in the provided blank spaces. When you have completed your work, save it as a pdf.

b) If you do not have a tablet with a stylus, but you do have access to a **printer**, print this pdf and write your answers to the exercises in the provided blank spaces. When you have completed your work, scan it with a printer or with a smartphone (in the latter case, you will need a **scanner app**, I personally use *Tiny scanner*)

c) If you have neither a tablet, nor a printer, solve as usual these exercises on a separate sheet of paper. When you have completed your work, scan it with your smartphone (you will need a **scanner app**, I personally use *Tiny scanner*).

Once you have your pdf, please submit it on `Gradescope.com` under the assignment *The Last Homework*.

If you have any doubt about the submission process, please ask me (via the chat of MS Teams or via email) before proceeding.

As usual you can work on the exercises with your friends (or enemies!) but the final editing has to be yours. This homework has to be submitted **by Wednesday April 22 at 9:30 am**. The total number of points for this homework is 120 (there are 20 extra points), but your final score will be the minimum between your score and 110. The grade you will receive for this homework will count as a part of *Homework* component of the total grade (15%).

**Ex 1. [30 points total]** Consider the function

$$\begin{array}{rccc} f: & \mathbb{Z}\times\mathbb{Z} & \to & \mathbb{Z} \\ & (x,y) & \mapsto & x+2y \end{array}$$

**(1a)** (10 points) Prove that $f$ is <u>not</u> one-to-one.

**(1b)** (10 points) Describe all the pre-images of $0 \in \mathbb{Z}$.

**(1c)** (10 points) Prove that $f$ is onto $\mathbb{Z}$.

**Ex 2. [30 points total]** Consider the function

$$\begin{array}{rccc} g: & \mathbb{R}\setminus\{2\} & \to & \mathbb{R} \\ & x & \mapsto & \frac{1}{x-2}+1 \end{array}$$

**(2a)** (15 points) Prove that $g$ is one-to-one.

**(2b)** (15 points) Prove that $g$ is <u>not</u> onto $\mathbb{R}$.

**Ex 3. [20 points total]** Let $f : A \to B$ and $g : B \to C$. We define the *composite* of $f$ and $g$:

$$\begin{array}{rccc} g \circ f : & A & \to & C \\ & x & \mapsto & g(f(x)) \end{array}$$

**(3a)** (10 points) Prove that if $f$ is onto $B$ and $g$ is onto $C$, then $g \circ f$ in onto $C$.

**(3b)** (10 points) Give an example of functions $f$ and $g$ such that $f$ is one-to-one and $g \circ f$ is <u>not</u> one-to-one (you can chose your favorite domains and codomains). Justify your answer.

**Ex 4. [25 points total]**

**(4a)** (10 points) Let $k \in \mathbb{Z}$. Prove that $k(k+1)$ is even.

**(4b)** (15 points) Prove that for all $n \in \mathbb{N}$, $6 \mid (n^3 + 5n)$.

**Ex 5. [15 points total]** For a natural integer $n \geq 2$, define

$$r_n := \underbrace{\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}}_{n \text{ times}}$$

For instance $r_4 = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1}}}}$.

**(5a)** (5 points) Write $r_{n+1}^2$ in function of $r_n$.

**(5b)** (10 points) Prove that for all natural integers $n \geq 2$, $r_n \notin \mathbb{Q}$.