Bridge - MGF 3301 - Section 001

Homework 10

Instructions

Please read and follow this instructions carefully, otherwise we will not be able to grade your work.

This homework contains 3 exercises. Keep in mind that at the end **you will have to submit a pdf** (no .png, .jpg, etc.). For that you will have three options:

a) If you have a **tablet with a stylus**, write your answers to the exercises directly on this pdf, in the provided blank spaces. When you have completed your work, save it as a pdf.

b) If you do not have a tablet with a stylus, but you do have access to a **printer**, print this pdf and write your answers to the exercises in the provided blank spaces. When you have completed your work, scan it with a printer or with a smartphone (in the latter case, you will need a **scanner app**, I personally use *Tiny scanner*)

c) If you have neither a tablet, nor a printer, solve as usual these exercises on a separate sheet of paper. When you have completed your work, scan it with your smartphone (you will need a **scanner app**, I personally use *Tiny scanner*).

Once you have your pdf, please submit it on `Gradescope.com` under the assignment *Homework 10*.

If you have any doubt about the submission process, please ask me (via the chat of MS Teams or via email) before proceeding.

As usual you can work on the exercises with your friends (or enemies!) but the final editing has to be yours. This homework has to be submitted **by Wednesday April 15 at 9:30 am**. The total number for this homework is 110 (there are 10 extra points). The grade you will receive for this homework will count as a part of *Homework* component of the total grade (15%).

Ex 1. [35 points total] The exercices here below are both about *partitions*, but they are independent of each other.

(1a) (10 points) Give an example of partition on the set $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

(1b) (25 points) Prove that

$$\mathscr{P} = \{[n, n+1) : n \in \mathbb{Z}\}$$

is a partition of $\mathbb{R}$, i.e. prove that:

- $\forall\, A \in \mathscr{P}, A \neq \varnothing$;
- $\forall\, A, B \in \mathscr{P}$, if $A \neq B$, then $A \cap B = \varnothing$;
  (*You may use the fact that if* $n, m \in \mathbb{Z}$ *such that* $n < m$, *then* $n+1 \leq m$.)
- $\displaystyle\bigcup_{n \in \mathbb{Z}} [n, n+1) = \mathbb{R}$.
  (*You may use the following notation: if* $x \in \mathbb{R}$ *then the* floor *of a real number* x, *denoted by* $\lfloor x \rfloor$, *is the largest integer that is less or equal than* x.)

Ex 2. [45 points total] Consider the set

$$X := \mathbb{Z} \times (\mathbb{Z} \setminus \{0\}) := \{(a, b) : a, b \in \mathbb{Z}, b \neq 0\}.$$

and the following relation on X:

$$(a, b) \sim (c, d) \Leftrightarrow ad - bc = 0.$$

(2a) (25 points) Prove that R is an equivalence relation on X. (Reflexivity and symmetry are almost straightforward. Transitivity is slightly more involved. Try to be creative and remember that the second coordinate of any element of X is different than 0).

(2b) (15 points) Describe the following equivalence classes using a set-builder notation:

2b.1) $\overline{(0,1)} =$

2b.2) $\overline{(1,1)} =$

2b.3) $\overline{(1,3)} =$

(2c) (5 points) The set of all equivalence classes of X modulo $\sim$, can be taken as the definition of a well known set. Which one?

- ☐ $\mathbb{N}$
- ☐ $\mathbb{Z}$
- ☐ $\mathbb{Q}$
- ☐ $\mathbb{R}$

Justify your answer:

Ex 3. [30 points total] Let $\mathbb{Z}_4 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}\}$ be the set of all equivalence classes for the relation congruence modulo 4. Recall that:

$$\overline{0} = \{4k : k \in \mathbb{Z}\}$$
$$\overline{1} = \{4k + 1 : k \in \mathbb{Z}\}$$
$$\overline{2} = \{4k + 2 : k \in \mathbb{Z}\}$$
$$\overline{3} = \{4k + 3 : k \in \mathbb{Z}\}$$

(3a) (15 points) Prove that if $x, y \in \overline{2}$ then $x + y \in \overline{0}$.

(3b) (15 points) Prove that if $x \in \overline{2}$ and $y \in \overline{3}$ then $x \cdot y \in \overline{2}$.