

Bridge - MGF 3301 - Section 001

Homework 1

Instructions: Solve the following exercises in a **separate sheet of paper**. Be tidy and organized! You can work on the exercises with your friends (or enemies!) but the final editing has to be yours. This homework has to be returned **by Wednesday January 22 at 9:30 am**. The total number for this homework is 110 (there are 10 extra points). The grade you will receive for this homework will count as a part of *Homework* component of the total grade (15%).

Note: Answers that are not fully and properly justified will not receive full credits.

Ex 1. (30 points) Diego, a bridge student, says:

“In summer 2020 I will either go to Italy and eat an authentic Neapolitan pizza, or attend a Research Experiences for Undergraduates (REU) program in California.”

- (1.1) Translate the above sentence into a propositional form, using upper case letters (specifying for each which simple proposition it represents), logical connectives and, if needed, parenthesis.
- (1.2) Reading the future, we can see that in summer 2020 Diego went to Italy, he did not eat an authentic Neapolitan pizza and he attended a *Research Experiences for Undergraduates (REU)* program in California. Was he lying? Justify fully your answer.

Ex 2. (30 points) Consider the following propositions:

- ▶ $P := \frac{4}{5} > \frac{5}{6}$.
- ▶ $Q := \text{“An icosahedron is a polyhedron with 20 faces”}$.
- ▶ $R := \text{“There are no real numbers between 3.14 and } \pi \text{”}$.

Determine the truth value of the propositional form

$$\sim(((\sim P) \wedge R) \vee Q),$$

for the above propositions P , Q and R . Justify fully your answer.

Ex 3. (40+10 points)

- (a) Consider the following definition:

Definition

A **tautology** is a propositional form that is true for every assignment of truth values to its components.

Using a truth table, prove that the following propositional form is a tautology:

$$(P \wedge Q) \vee ((\sim P) \vee (\sim Q)). \tag{1}$$

- (b) Let A and B be two sets, and let x be an element of the union $A \cup B$. Let also:
 - ▶ $P := \text{“}x \text{ is an element of } A \text{”}$.
 - ▶ $Q := \text{“}x \text{ is an element of } B \text{”}$.

Now answer the following questions:

- (b1) In terms of set theory, what is the propositional form (1) saying in this particular case? You may use a Venn diagram to support your answer.
- (b2) Explain what it means for the propositional form (1) to be a tautology in this context.